

Here I, (Yuhei Suzuki, Associate Professor at Hokkaido University) report an error of the article arXiv:2505.00587v3 by Adam Dor-On and Ian Thompson, which claimed a solution to the reduced Hao-Ng isomorphism Problem. I hope this information is helpful to anyone who are interested in and/or want to use the reduced Hao-Ng isomorphism problem for appropriately and accurately evaluating their work as well as our contribution to the (rigorous and concise) solution to the reduced Hao-Ng problem (for C^* -dynamics).

The error occurs in Theorem 3.1, which plays a fundamental role in the proof of the Main Theorem (more precisely, in the key step 4.3).

Counterexamples are surprisingly easy to find. Indeed, if G is not discrete, take $B = C_0(G)$ equipped with the left translation action, and consider the map into $l^\infty(G)$, which is the direct summand of the second dual corresponding to the atomic measures.

This map is of course injective. However, since $l^\infty(G)$ has no nonzero G -continuous direct summand, cutting down by z_α makes the map zero, so the injectivity claimed there fails.

(Note(can be skipped to read): I myself dealt with this von Neumann algebra in my work [16] with Ozawa on amenable C^* -dynamical systems, and it (as well as the second dual) is an object with which it is extremely easy to make mistakes;

proofs involving it really must be carried out with the utmost care.

In fact, even in our paper on amenability, while we were making final revisions in preparation for its public release, we came across errors several times that required us to substantially rewrite the proofs.

Judging from the mistake I found this time, it does not appear to me that the authors have carried out the kind of careful verification that is necessary here.)

As I have not read the entire paper,

I cannot tell how far this error affects their isomorphism results (but I have checked that this incorrect claim is used in a fundamental way in 4.3, a key result).

Moreover, according to the authors' update notes on arXiv for v3, Theorem 3.1 was introduced precisely to fix a fatal flaw in the previous versions.

I am therefore concerned that this error may also be fatal to the present proof of the main theorem.

I have informed this error to the authors on September 25, 2026.