An imitation model based on the majority

Department of Applied Mathematics, National Sun Yat-sen University
Hsin-Lun Li *

Abstract

The voter model consists of a set of agents whose opinions are binary variables. At each time step, an agent, along with a randomly chosen social neighbor, is selected, and the agent imitates the social neighbor at the next time step. In this paper, we investigate a variant of the voter model known as an imitation model based on the majority. In this variant, an agent imitates the opponents' opinion if the number of social neighbors holding the opponents' opinion is greater than the number of social neighbors holding the same opinion as the agent. We examine the probability of achieving consensus on a finite connected social graph.

1 Introduction

In the voter model, an individual with binary opinions ± 1 is uniformly selected at each time step and imitates one of its randomly chosen social neighbors. Each individual has a probability of p to be $+1$. We consider a variant of the voter model in which an individual is uniformly selected at each time step and imitates opponents' opinions if the number of its social neighbors holding the opponent's opinion is greater than the number of its social neighbors holding the same opinion as it. The variant is called an imitation model based on the majority. Unlike the voter model, a consensus cannot always be achieved on a finite social graph for this variant. We discuss the variant on a connected social graph $G = ([n], E)$ with the vertex set and edge set $[n] = \{1, \dots, n\}$ and E.

2 Main Results

It turns out that there is almost surely a consensus on the complete social graph. Furthermore, $1 - 2p(1-p)|E|$ serves as a lower bound for the probability of consensus on any connected social graph G.

Theorem 1 *We have* $P(consensus) = 1$ *on the complete social graph and* $P(consensus) \geq 1 - 2p(1-p)|E|$ *on all connected social graphs* G.

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*E-mail:hsinlunl@math.nsysu.edu.tw

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