Transmuted Logistic II distribution

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Abstract

A generalization of the generalized inverse Logistic II distribution so-called transmuted generalized inverse Logistic II distribution is proposed and studied. The studies we have done: calculate the first and second time, set the survival function, risk function and a study of statistical order.

Keywords: transmuted, logistic II distribution, heavy-tailed distribution, asymmetry parameter.

Introduction

The models are an important tool for estimating the behavior of some population. However, in many cases, they show some problems of adjustment with the data. The Logistics II model describes a similar curve with normal, symmetrical model, but with a correction in tails. It has interesting applications in modeling chronic obstructive respiratory disease, geological issues, Hemolytic Uremic Syndrome Data (HUS), physical-chemical phenomena, psychological issues, the survival time of patients diagnosed with leukemia, and weight gain data. [2] In this paper we built a model transmuted Logistics II. In order to improve the model data that already known to have heavy tails characteristic of more asymmetrically. A transmuted a transformation model is made in a model in order to generalize it. This model has a parameter more, the asymmetry parameter.

Logistic II Model

The probability density function (pdf) of the logistic II distribution is given by:

$$f(x;\mu,\delta)=\frac{\exp\left\{-\left(\frac{x-\mu}{\delta}\right)\right\}}{\delta\left(1+\exp\left\{-\left(\frac{x-\mu}{\delta}\right)\right\}\right)^2} \tag{1}$$

at where $x\in\mathbb{R}^+$, $\mu\in\mathbb{R}$ e $\delta\in\mathbb{R}^+$.
We have the cdf of logistic II distribution:

$$F(x;\mu,\delta)=\int_{-\infty}^{x}\frac{\exp\left\{-\left(\frac{t-\mu}{\delta}\right)\right\}}{\delta\left(1+\exp\left\{-\left(\frac{t-\mu}{\delta}\right)\right\}\right)^2}dt=\frac{1}{1+\exp\left\{-\left(\frac{x-\mu}{\delta}\right)\right\}} \tag{2}$$

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Transmuted Model

This section explains how the transmuted model is constructed of any existing model, and F_1 a model of the distribution function known and f_2 to the new model. The new model is defined by the following transformation: [1]

$$F_2(x) = (1+\lambda)F_1(x) - \lambda F_1(x)^2 \tag{3}$$

which on differentiation yields,

$$f_2(x) = f_1(x)[(1+\lambda) - 2\lambda F_1(x)] \tag{4}$$

for $\lambda \in [-1,1]$ where $f_1(x)$ e $f_2(x)$ are the pdfs of $F_1(x)$ and $F_2(x)$.

Transmuted Logistic II Model

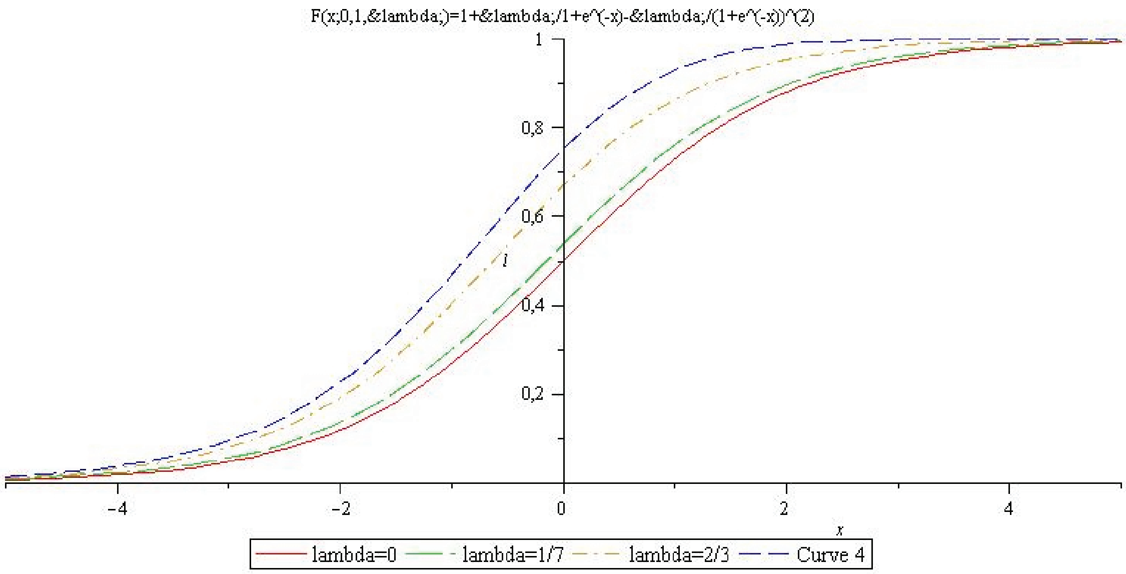
In this section we use the results shown in the previous section in the logistic II model and thus create a new model. Applying equation 2 the equation 4 define the density function:

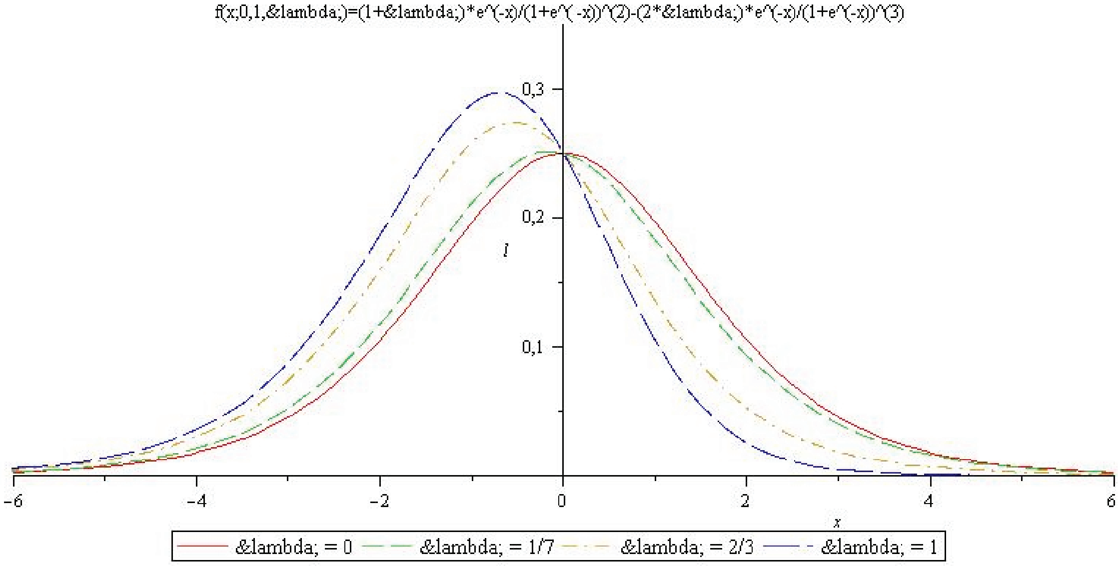
$$F_{TL2}(x;\mu,\delta,\lambda) = \frac{1}{1+\exp\left\{-\left(\frac{x-\mu}{\delta}\right)\right\}}\left[(1+\lambda) - \frac{\lambda}{\left(1+\exp\left\{-\left(\frac{x-\mu}{\delta}\right)\right\}\right)}\right]$$

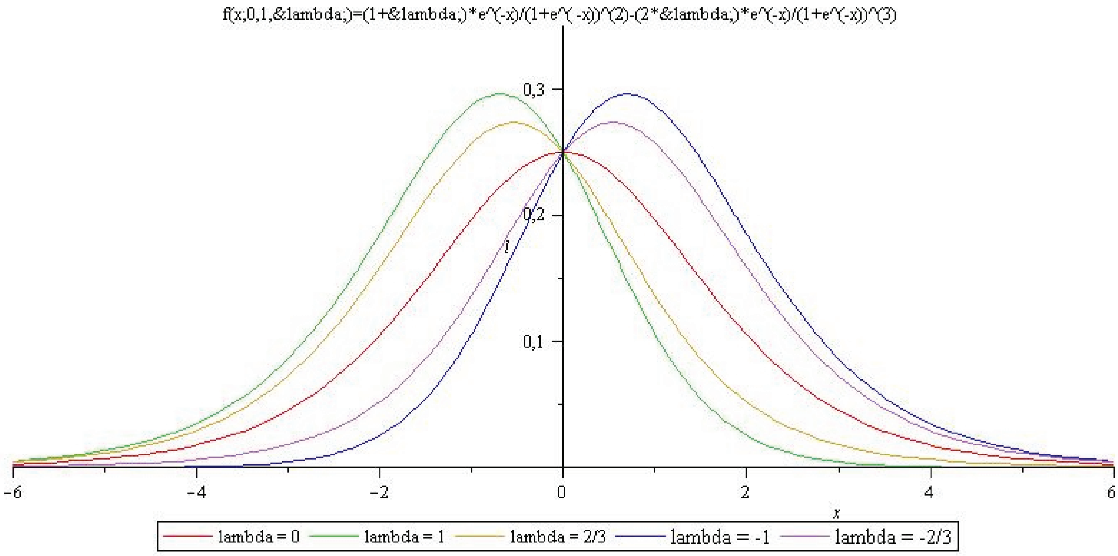
Also we define the probability density function of the logistic model transmuted II:

$$f_{TL2}(x;\mu,\delta,\lambda) = \frac{\exp\left\{-\left(\frac{x-\mu}{\delta}\right)\right\}}{\delta\left(1+\exp\left\{-\left(\frac{x-\mu}{\delta}\right)\right\}\right)^2}\left\{(1+\lambda) - \frac{2\lambda}{1+\exp\left\{-\left(\frac{x-\mu}{\delta}\right)\right\}}\right\}$$

for $\lambda \in [-1,1]$.







The first and second figures above are some examples of distribution functions and pdf for some values of λ. The third figure shows the asymmetry in cases where comparing two constants and their respective opposite to zero and constant, if there is symmetry in the model.

Regardless of the values μ and δ we can standardize this model as a random variable $Z = \frac{X-\mu}{\delta} \sim TLog2(Z)$ to facilitate the bills. Thus we have the distribution function:

$$F_{TL2}(z) = \frac{1}{1+\exp\{-z\}}\left[(1+\lambda) - \frac{\lambda}{1+\exp\{-z\}}\right]$$

and

$$f_{TL2}(z) = \frac{\exp\{-z\}}{(1+\exp\{-z\})^2}\left\{(1+\lambda) - \frac{2\lambda}{1+\exp\{-z\}}\right\}$$

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