The lace expansion for the nearest-neighbor models on the BCC lattice

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Abstract

The lace expansion is a powerful tool to analyze the critical behavior in high dimension. It is applied to various stochastic models, such as self-avoiding walk, percolation, etc. By using the lace expansion, we want to prove that the nearest-neighbor self-avoiding walk and percolation exhibit the mean-field behavior in dimension d above the upper critical dimension $d_\mathrm{c} = 4$, 6, respectively. In particular, our purpose is to prove them in beginner-friendly manner. To do so, we used the d-dimensional body-centered cubic lattice $\mathbb{L}^d$. As a result, we have confirmed that the nearest-neighbor self-avoiding walk on $\mathbb{L}^{d \geq 6}$ and percolation on $\mathbb{L}^{d \geq 9}$ exhibit the mean-field behavior. This is a little worse for the self-avoiding walk but better for the percolation than previous research (cf. the self-avoidng walk on $\mathbb{Z}^{d \geq 5}$, the percolation on $\mathbb{Z}^{d \geq 11}$). Moreover, the proof is relatively more simple.

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