7.62. If $\mathbf{A} \times \mathbf{B} = \mathbf{A} \times \mathbf{C}$, does $\mathbf{B} = \mathbf{C}$ necessarily?

7.63. Find the area of the triangle with vertices (2, –3, 1), (1, –1, 2), (–1, 2, 3).

Ans. $\frac{1}{2}\sqrt{3}$

7.64 Find the shortest distance from the point (3, 2, 1) to the plane determine by (1, 1, 0), (3, –1, 1), (–1, 0, 2).

Ans. 2

Triple products

7.65. If $\mathbf{A} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$, $\mathbf{B} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{C} = -\mathbf{i} + \mathbf{j} - 4$, find (a) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$, (b) $\mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$, (c) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$, and (d) $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$.

Ans. (a) 20 (b) 20 (c) $8\mathbf{i} - 19\mathbf{j} - \mathbf{k}$ (d) $25\mathbf{i} - 15\mathbf{j} - 10\mathbf{k}$

7.66. Prove that (a) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$ and (b) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}\,(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$.

7.67. Find an equation for the plane passing through (2, –1, –2), (–1, 2, –3), (4, 1, 0).

Ans. $2x + y - 3z = 9$

7.68. Find the volume of the tetrahedron with vertices at (2, 1, 1), (1, –1, 2), (0, 1, –1), (1, –2, 1).

Ans. $\frac{4}{3}$

7.69. Prove that $(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) + (\mathbf{B} \times \mathbf{C}) \cdot (\mathbf{A} \times \mathbf{D}) + (\mathbf{C} \times \mathbf{A}) \cdot (\mathbf{B} \times \mathbf{D}) = 0$.

Derivatives

7.70 A particle moves along the space curve $\mathbf{r} = e^{-t} \cos t\mathbf{i} + e^{-t} \sin t\mathbf{j} + e^{-t}\mathbf{k}$. Find the magnitude of the (a) the velocity and (b) the acceleration at any time t.

Ans. (a) $\sqrt{3}e^{-1}$ (b) $\sqrt{5}e^{-1}$

7.71. Prove that $\frac{d}{du}(\mathbf{A} \times \mathbf{B}) = A \times \frac{d\mathbf{B}}{du} + \frac{d\mathbf{A}}{du} \times \mathbf{B}$ where **A** and **B** are differentiable functions of u.

7.72. Find a unit vector tangent to the space curve $x = t$, $y = t^2$ $z = t^3$ at the point where $t = 1$.

Ans. $(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})/\sqrt{14}$

7.73. If $\mathbf{r} = \mathbf{a}\cos \omega t + \mathbf{b} \sin \omega t$, where **a** and **b** are any constant noncollinear vectors and ω is a constant scalar. prove that (a) $\mathrm{r} = \frac{d\mathbf{r}}{dr} = \omega(\mathrm{a} \times \mathrm{b})$ and (b) $\frac{d^2\mathbf{r}}{dt^2} + \omega^2\mathbf{r} = 0$.

7.74. If $\mathbf{A} = x^2\mathbf{i} - y\mathbf{j} + xz\mathbf{k}$, $\mathbf{B} = y\mathbf{i} + x\mathbf{j} - xyz\mathbf{k}$, and $\mathbf{C} = \mathbf{i} - y\mathbf{j} + x^3 z\mathbf{k}$, find (a) $\frac{\partial^2}{\partial x\, \partial y}(\mathbf{A} + \mathbf{B})$ and (b) $d[\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})]$ at the point (1, –1, 2).

Ans. (a) $-4\mathbf{i} + 8\mathbf{j}$ (b) $8\,dx$