Calculus 1: Final Exam (Sample)

Solve, justifying your answers, the following exercises at home. We will correct in class next thursday.

Exercise 1. Let $\mathcal{C}$ the curve in the space parametrized by

$$\begin{aligned} f : &\mathbb{R} \longrightarrow \mathbb{R}^3 \\ &t \longmapsto (t^3 - \sin 2t, te^t, t^2 - \cos t) \end{aligned}$$

1. compute velocity and acceleration of a particle moving along the curve $\mathcal{C}$ at time $t = 0$; (10)
2. compute the tangent line to $\mathbb{C}$ at $t = \frac{\pi}{2}$; (7.5)
3. compute the normal plane to $\mathbb{C}$ at $t = \frac{\pi}{2}$. (7.5)

Exercise 2. (25) Find maxima, minima and saddle point of the function $z = f(x, y)$ defined by:

$$f(x, y) = x^2 + y^4 + 2y^2 - 4xy + 1$$

Exercise 3. Consider the surface $\mathcal{S}$ defined by $x^3y^2z - 3z^2 = 4xy - 5z + 2x$.

1. Find the tangent plane to $\mathcal{S}$ at the point (1,1,0); (7.5)
2. find the normal line to $\mathcal{S}$ at the point (1,1,0); (7.5)
3. find the tangent line at the point (1,1,0) to the level curve $\mathcal{C}$ obtained intersecting $\mathcal{S}$ with the plane π defined by the equation $z = 0$. (10)

Exercise 4. Consider the two variables function defined by $f(x, y) = \log(\sqrt{x^2 + y^2})$.

1. Compute the domain of f; (5)
2. compute the Laplacian $\Delta f(x, y) = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$; (5)
3. consider polar coordinates $x = r\cos\theta$, $y = r\sin\theta$ $(r > 0)$. By the chain rule, compute the partial derivatives $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial \theta}$ in terms of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$; (5)
4. show that for any function $z = f(x, y)$ the following equality holds:(10)

$$(\frac{\partial f}{\partial r})^2 + \frac{1}{r}(\frac{\partial f}{\partial \theta})^2 = (\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y}).$$