Applications

Relative Extrema and Points of Inflection

See Chapter 3, where relative extrema and points of inflection are described and a diagram is presented. In this chapter such points are characterized by the variation of the tangent line and then by the derivative, which represents the slope of that line.

Assume that f has a derivative at each point of an open interval and that P_1 is a point of the graph of f associated with this interval. Let a varying tangent line to the graph move from left to right through P_1. If the point is a relative minimum, then the tangent line rotates counterclockwise. The slope is negative to the left of P_1 and positive to the right. At P_1 the slope is zero. At a relative maximum a similar analysis can be made except that the rotation is clockwise and the slope varies from positive to negative. Because f'' designates the change of f', we can state the following theorem. (See Figure 4.6.)

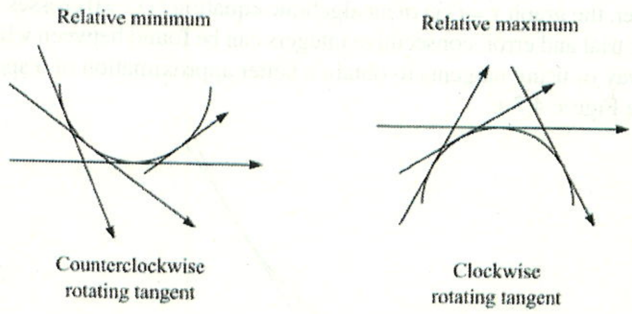

Figure 4.6

Theorem Assume that x_1 is a number in an open set of the domain of f at which f' is continuous and f'' is defined. If $f'(x_1) = 0$ and $f''(x_1) \neq 0$, then $f(x_1)$ is a relative extreme of f. Specifically:

(a) If $f''(x_1) > 0$, then $f(x_1)$ is a relative minimum.

(b) If $f''(x_1) < 0$, then $f(x_1)$ is a relative maximum.

(The domain value x_1 is called a *critical value*.)

This theorem may be generalized in the following way. Assume existence and continuity of derivatives as needed and suppose that $f'(x_1) = f''(x_1) = \ldots f^{(2p-1)}(x_1) = 0$ and $f^{(2p)}(x_1) \neq 0$ (p a positive integer). Then:

(a) f has a relative minimum at x_1 if $f^{(2p)}(x_1) > 0$.

(b) f has a relative maximum at x_1 if $f^{(2p)}(x_1) < 0$.

(Notice that the order of differentiation in each succeeding case is two greater. The nature of the intermediate possibilities is suggested in the next paragraph.)

It is possible that the slope of the tangent line to the graph of f is positive to the left of P_1, zero at the point, and again positive to the right. Then P_1 is called a *point of inflection*. In the simplest case this point of inflection is characterized by $f'(x_1) = 0$, $f''(x_1) = 0$, and $f'''(x_1) \neq 0$.

Particle Motion

The fundamental theories of modern physics are relativity, electromagnetism, and quantum mechanics. Yet Newtonian physics must be studied because it is basic to many of the concepts in these other theories, and because it is most easily applied to many of the circumstances found in everyday life. The simplest aspect of Newtonian mechanics is called *kinematics*, or the *geometry of motion*. In this model of reality, objects are idealized as points and their paths are represented by curves. In the simplest (one-dimensional) case, the curve is a straight line, and it is the speeding up and slowing down of the object that is of importance. The calculus applies to the study in the following way.

If x represents the distance of a particle from the origin and t signifies time, then $x = f(t)$ designates the position of a particle at time t. Instantaneous velocity (or speed in the one-dimensional case) is represented