Arithmetic and Algebraic Geometry 2013

Organizers: G. van der Geer (Univ. Amsterdam)
T. Katsura (Hosei Univ.)
I. Nakamura (Hokkaido Univ.)
T. Terasoma (Univ. Tokyo)

Dates : January 28 (Monday)–January 31 (Thursday), 2013
Place: Graduate School of Mathematical Sciences
The University of Tokyo

Program:

January 28 (Monday)

10:00–11:00 : Shigeru Mukai (RIMS)
Enriques surfaces and Igusa quartic

11:20–12:20 : Fumiharu Kato (Kumamoto Univ.)
TBA

13:40–14:40 : Gerard van der Geer (Univ. Amsterdam)
Modular forms for genus 2 and 3

15:10–16:10 : Kentaro Mitsui (Kobe Univ.)
Homotopy exact sequences for fibrations with non-reduced geometric fibers

16:30–17:30 : Takehiko Yasuda (Osaka Univ.)
Stringy invariants of p-cyclic quotient singularities

January 29 (Tuesday)

10:00–11:00 : Eyal Goren (McGill Univ.)
On a conjecture of Bruinier-Yang

11:20–12:20 : Ichiro Shimada (Hiroshima Univ.)
Supersingular K3 surfaces with Artin invariant 10

13:40–14:40 : Tetsuji Shioda (Rikkyo Univ./RIMS)
Manin's map and Mordell-Weil Lattices

15:10–16:10 : Hisanori Ohashi (Tokyo Univ. Sci.)
On automorphisms of Enriques surfaces

16:30–17:30 : Matthias Schuett (Leibniz Univ. Hannover)
Picard numbers of quintic surfaces

18:00: Reception

January 30 (Wednesday)

10:00–11:00 : Lin Weng (Kyushu Univ.)
  Higher rank zeta functions and Riemann hypothesis for elliptic curves

11:20–12:20 : Ching-Li Chai (Univ. Pennsylvania)
  The Hecke orbit problem and local symmetries

13:40–14:40 : Kazuhiro Fujiwara (Nagoya Univ.)
  Non-abelian class field theory and indivisibility of class numbers of number fields

15:10–16:10 : Shinichi Kobayashi (Tohoku Univ.)
  A p-adic approach to the Birch and Swinnerton-Dyer conjecture

16:30–17:30 : Masato Kurihara (Keio Univ.)
  Refined Iwasawa theory and refined Birch Swinnerton-Dyer conjecture

January 31 (Thursday)

10:00–11:00 : Guido Kings (Regensburg Univ.)
  TBA

11:20–12:20 : Iku Nakamura (Hokkaido Univ.)
  The closed immersions of the compactification $SQ^{\mathrm{toric}}_{g,K}$ into Alexeev's complete moduli $\overline{AP}_{g,N}$

13:40–14:40 : Tomohide Terasoma (Univ. Tokyo)
  Brown-Zagier identity for associator

15:00–16:00 : Shuji Saito (Tokyo Inst. Tech.)
  Existence conjecture for smooth sheaves on varieties over finite fields

Arithmetic and Algebraic Geometry 2013

科学研究費補助金基盤研究(S)No.23224001（代表者：中村郁），基盤研究(S) No.24224001（代表者：斎藤政彦），東京大学Global COE「The Research and Training Center for New Development in Mathematics」（リーダー：川又雄二郎）
により、標記のような研究集会を下記の要領で開催しますのでご案内致します。

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ABSTRACTS OF LECTURES
ARITHMETIC ALGEBRAIC GEOMETRY 2013
JANUARY 28 - 31, UNIVERSITY OF TOKYO

Ching-Li Chai (University of Pennsylvania, USA)

The Hecke orbit problem and local symmetries.

Abstract. The action of the local stabilizer subgroup on the formal completion of a closed point of the reduction of a Shimura variety holds much information about the Hecke symmetry. This action goes back to Lubin and Tate but is not well-understood. We will explain a method, which traces back to ideas of Taira Honda, to compute this action integrally.

Kazuhiro Fujiwara (Nagoya University, Japan)

Non-abelian class field theory and indivisibility of class numbers of number fields.

Abstract. For a totally real field F and an odd prime p, we look for a CM quadratic extension L/F whose relative class number is not divisible by p. This question of indivisibility is classical, and turns out to be fairly difficult. I show the existence of infinitely many L by using modularity and Galois deformations, namely, in the framework of non-abelian class field theory. This is part of my project on non-abelian approaches to classical problems. An application of this indivisibility result will be given at another occasion.

Fumiharu Kato (Kumamoto University, Japan)

Yet another Fake Projective Plane via p-adic uniformization

Abstract. This is a joint-work with Daniel Allcock (Austin). A fake projective plane is a smooth complex algebraic surface that has the same Betti numbers as, but is not isomorphic to, the projective plane. The first example has been constructed by Mumford more than 30 years ago, which has been yielded from 2-adic uniformization. Ishida-Kato showed that there are two more FPP's coming from 2-adic uniformization. A complete classification of FPP's is nowadays known due to recent works by Prasad-Yeung and Cartwright-Steger. It is also known that the three first examples, Mumford's and Ishida-Kato's, are the only possible FPP's that can be obtained from 2-adic uniformization. In this talk, we will see that there's yet another FPP that can be constructed in the framework of 2-adic uniformization; namely, by quotient of Drinfeld ball by a uniform lattice with torsions. In order to show that the new FPP is not isomorphic to the previously known ones, we go to the associated Berkovich spaces, and compare the homotopy type of them.

Eyal Goren (McGill University, Canada)

On a conjecture of Bruinier-Yang.

Abstract. I will describe significant progress towards proving a conjecture of Bruinier-Yang concerning intersection of CM points and special divisors on Shimura varieties of orthogonal type. The conjecture is a vast generalization of the theorem of Gross-Zagier on singular moduli. This is joint work with Andreatta (Milano), Howard (Boston College) and Madapusi-Pera (Harvard).

Shin-ichi Kobayashi (Tohoku University, Japan)

A p-adic approach to the Birch and Swinnerton-Dyer conjecture.

Abstract. The famous Birch and Swinnerton-Dyer conjecture (BSD conjecture) is a formula relating the Mordell-Weil rank of elliptic curve E over $\mathbf{Q}$ to the order of the Hasse-Weil L-function of E over $\mathbf{Q}$ at $s = 1$. There is also a strong version that relates the leading term of the L-function to arithmetic invariants of E such as the order of the Tate-Shafarevich group.

In this talk, we explain recent progress on the strong BSD conjecture for the rank 1 case via p-adic methods such as the Iwasawa main conjecture and the p-adic Gross-Zagier formula. Our approach is based on a mysterious relation between the BSD conjcture and the p-adic BSD conjecture. Usually, the p-adic conjecture is considered as just a p-adic analogue of the complex one, and there is no logical link ;between them except the rank zero case.

However, the complex and p-adic Gross-Zagier formula relates these conjectures directly in the rank 1 case. Since we know the p-adic BSD conjecture much better than the complex conjecture by K. Rubin and K. Kato for the Iwasawa main conjecture (and also by a recent work of Skinner-Urban), we obtain complex results from p-adic results in the rank one case.

Shigeru Mukai (RIMS, Japan)

Enriques surfaces and Igusa quartic

Abstract. An Enriques surface is the quotient of an (algebraic) K3 surface by a fixed-point-free involution. By Torelli, their moduli space is a 10-dimensional orthogonal modular variety. More generally, for a negative definite integral quadratic lattice L and its embedding into $I_{2,10}$, one obtains an algebraic family of L-polarized Enriques surfaces parametrized by a $(10 - \mathrm{rk}\, L)$-dimensional orthogonal modular variety, together with the discriminant locus where the above involutions acquire fixed points. Here $I_{2,10}$ denotes the odd unimodular lattice of sugnature $(2+, 10-)$. One can study various algebraic/arithmetic geometry for this family. In this talk I will discuss several samples, including two cases where L is the root lattice of type D_6+A_1 and of type E_7. In the former case an L-polarized Enriques surface is covered by a Jacobian Kummer surface and the 3-dimensional orthogonal modular variety has Igusa quartic as its Satake compactification. I will also discuss E_7-polarized Enriques surfaces and their modular invariants if time permits.

Iku Nakamura (Hokkaido University, Japan)

The closed immersions of the compactification $SQ_{g,K}^{toric}$ into Alexeev's complete moduli $\overline{AP}_{g,N}$.

Abstract. There are three relevant moduli spaces, that is, two geometric compactifications $SQ_{g,K}$ (N1999) and $SQ_{g,K}^{\mathrm{toric}}$ (N2010) of the moduli of abelian varieties, and Alexeev's complete moduli $\overline{AP}_{g,d}$ of generalized abelian varieties, each with a semiabelian group action and an ample divisor (Alexeev2002). In this talk, we review the known results about these separated complete moduli spaces. Then, we compare our second compactification $SQ_{g,K}^{\mathrm{toric}}$, and Alexeev's complete moduli $\overline{AP}_{g,N}$. We prove

(i) if $|K| = N^2$, there is a $(N-1)$-dimensional effective family over $\mathbf{Z}[\zeta_N, 1/N]$ of *closed immersions* of $SQ_{g,K}^{\mathrm{toric}}$ into $\overline{AP}_{g,N}$,

(ii) $SQ_{g,1}^{\mathrm{toric}} \simeq \overline{AP}_{g,1}$.

Matthias Schuett (Leibniz Universität Hannover, Germany)

Picard numbers of quintic surfaces.

Abstract. The Picard number is a non-trivial invariant of an algebraic surface which captures much of its inner structure. We will discuss the fundamental problem which Picard numbers occur on surfaces on general type for the prototype example of quintics in P3. We will review what seems to be known and introduce a new technique based on arithmetic deformations which allows us to engineer quintics with prescribed Picard number.

Shuji Saito (Tokyo Institute of Technology)

Existence conjecture for smooth sheaves on varieties over finite fields.

Abstract. This is a joint work with Moritz Kerz. Let X be a smooth variety over a finite field $\mathbf{F}_q$. For an integer $r > 0$, let $\mathcal{S}_r(X)$ be the set of lisse $\overline{\mathbf{Q}_\ell}$-sheaves on X of rank r up to isomorphism and up to semi-simplification. Let $Cu(X)$ be the set of normalizations of integral curves on X. Let $\mathrm{SK}_r(X)$ be the set of systems $(V_Z)_{Z\in Cu(X)}$ with $V_Z \in \mathcal{S}_r(Z)$ such that

$$(V_Z)_{|Z\times_X Z'} = (V_{Z'})_{|Z\times_X Z'} \quad \text{for } Z, Z' \in Cu(X).$$

The question is how to determine the image of the restriction map

$$\tau : \mathcal{S}_r(X) \to \mathcal{S}k_r(X),$$

i.e. when a system $(V_Z)_{Z\in Cu(X)}$ glues to a lisse $\overline{\mathbf{Q}_\ell}$-sheaf on X. We explain a conjecture of Deligne on the problem which describes the image in terms of a ramification condition at infinity and prove the conjecture in case $r = 1$.

Ichiro Shimada (Hiroshima University, Japan)

Supersingular K3 surfaces with Artin invariant 10
(joint work with S. Kondo)

Abstract. We classify genus one fibrations on supesingular K3 surfaces with Artin invariant 10 in characteristic 2 and 3, and investigate the automorphism group of these supersingular K3 surfaces.

Tetsuji Shioda (Rikkyo University/RIMS, Japan)
Manin's map and Mordell-Weil Lattices.

Abstract. Given an elliptic surface (or an elliptic curve E/K over a function field), Manin defines a group homomorphism from the Mordell-Weil goup E(K) to K (the additive group) in terms of the Picard-Fuchs equation, which is injective modulo torsion. We compute some concrete examples of Manin's map using the idea of excellent families, etc.

Gerard van der Geer (Amsterdam University, Netherlands)
Modular Forms of Genus Two and Level Two.

Abstract. We present work in progress with Fabien Clery and Sam Grushevsky on vector-valued Siegel modular forms of degree 2 and level 2.

Lin Weng (Kyushu University, Japan)
Higher rank zeta functions and Riemann hypothesis for elliptic curves.

Abstract. For an irreducible, reduced, regular projective curve X over a finite field $\mathbf{F}_q$ and a fixed integer $n \geq 1$, we define the rank n pure non-abelian zeta function of $X/\mathbf{F}_q$ by

$$\zeta_{X/\mathbf{F}_q,n}(s) = \sum_{[V]} \frac{|H^0(X,V)| - 1}{|\operatorname{Aut}(V)|}\, q^{-\deg(V)\, s} \qquad (\Re(s) > 1),$$

where the sum is over isomorphism classes of $\mathbf{F}_q$-rational semi-stable vector bundles V of rank n on X with degree divisible by n. This function, which agrees with the usual Artin zeta function of $X/\mathbf{F}_q$ if $n = 1$, is a rational function of q^{-s} with the standard functional equation, and conjecturally satisfies the Riemann hypothesis. In this talk we report our recent joint work with Zagier on these zetas for elliptic curves $E/\mathbf{F}_q$. With the help of the so-called special and general counting miracles, we show that the Dirichlet series

$$\mathfrak{Z}_{E/\mathbf{F}_q}(s) = \sum_{[V]} \frac{1}{|\operatorname{Aut}(V)|}\, q^{-\operatorname{rank}(V)s} \qquad (\Re(s) > 0),$$

where the sum is now over isomorphism classes of $\mathbf{F}_q$-rational semi-stable vector bundles V of degree 0 on X, is equal to $\prod_{k=1}^{\infty} \zeta_{E/\mathbf{F}_q}(s+k)$, and use this fact to deduce the validity of the Riemann hypothesis for $\zeta_{E,n}(s)$ for all n.

Takehiko Yasuda (Osaka University, Japan)
Stringy invariants of p-cyclic quotient singularities

Abstract. This talk will be concerned with quotient singularities associated with modular representations of a cyclic group of prime order. I will explain how generalized motivic integration enables us to compute stringy invariants of such singularities in terms of the representation. Motivic integration is generalized to relevant wild quotient stacks. Then the stringy invariant of the quotient variety is expressed as some motivic integral on the space of twisted arcs of the quotient stack, and related to some weighted count of Artin-Schreier extensions of the power series field.