ABSTRACTS OF LECTURES
RECENT PROGRESS IN NONKÄHLER GEOMETRY
MARCH 5 - 7, HOKKAIDO UNIVERSITY

Georges Dloussky (CMI Universite d'Aix-Marseille, France)
VII-class surfaces with a cycle of rational curves and strongly bihermitian surfaces

Abstract. When there exists a m-numerical anticanonical divisor the surface contains a GSS. This result will be applied to end the classification of strongly bihermitian surfaces (joint work with V. Apostolov). We shall also consider the weaken condition of surfaces with a cycle of rational curves.

Serguei Ivashkovich (Lille University, France)
Foliated shells as obstructions to the existence of skew cylinders for non-Kähler foliations

Abstract. The object known as a *skew cylinder*, introduced by Ilyashenko is proved to be an extremely useful tool in foliation theory. Existence of skew cylinders for 1-dimensional holomorphic foliations on Stein manifolds was proved by Ilyashenko himself and on Kähler manifolds by Brunella in (under the name of a *covering tube*). We give an explicit obstruction for the existence of skew cylinders for holomorphic 1-dimensional foliations on the wide class of non-Kähler manifolds (including all compact complex surfaces for example). We call this object a *foliated shell*. A number of related statements will be given and several open questions will be discussed.

Finnur Larusson (University of Adelaide, Australia)
Generalisations of Trépreau's lemma on schlicht envelopes of holomorphy

Abstract. I will report on work in progress with Rasul Shafikov of the University of Western Ontario in Canada, concerned with the following basic question. Let Y be a domain in a complex manifold X. Is there a largest Stein domain Z in X containing Y to which all holomorphic functions on Y extend, and if so, what can we say about Z?

Suppose $\mathcal{F}$ is a smooth 1-dimensional foliation of X. We call a domain Y in X an *interval domain* with respect to $\mathcal{F}$ if Y has nonempty connected intersection with every noncompact leaf of $\mathcal{F}$, and Y contains the closure of the union of the compact leaves of $\mathcal{F}$.

Our main result is that if Y is an interval domain and $\mathcal{F}$ has a property, apparently not defined before, that we call *quasiholomorphicity*, then there is a largest interval domain Z containing Y to which all holomorphic functions on Y extend. Moreover, Z is locally Stein. Hence, if X is Stein (or under some other conditions), we can conclude that Z is Stein. This generalises a lemma of Trépreau (Invent. Math. 1986).

I will introduce the notion of quasiholomorphicity and illustrate it with examples, describe the proof of the main result and variants and special cases of it, and discuss some applications.

Iku Nakamura (Hokkaido University, Japan)
Survey on VII_0 surfaces

Abstract. I will quickly recall some of the basic facts about surfaces of class VII_0 with positive second Betti number. This will be an introduction to the lectures of Professors Dloussky and Teleman.

Andrei Teleman (1)-(4) (CMI, LATP, Universite de Provence, France)

(1) Moduli space of stable holomorphic bundles over non- Kahler surfaces
(2) Existence of curves on class VII surfaces with small b_2
(3) (March 7 Seminar) The canonical extension
(4) (March 7 Seminar) Families of holomorphic bundles

Abstract of (1). We introduce the concept of stable bundle over a surface endowed with a Gauduchon metric, and we show that the set of isomorphism classes of stable holomorphic structures on a fixed differentiable bundle has a natural structure of Hausdorff complex space (called the moduli space of stable structures). We explain the Kobayashi-Hitchin correspondence in the general non-Kahlerian framework; this fundamental result allows us to identify a moduli space of stable bundles with a moduli space of irreducible instantons. The Kobayashi- Hitchin correspondence was first used in classical Donaldson theory as a tool to describe moduli spaces of instantons on a complex surface, and to compute explicitly Donaldson polynomial invariants. Finally we apply the general theory to a particular moduli space of bundles over a minimal class VII surface with $b_2 \in \{1, 2\}$, and we point out the first geometric properties (local structure, compactness, symmetry) of this moduli space.

Abstract of (2). We explain our general program to prove existence of curves on minimal class VII surfaces using moduli spaces of polystable holomorphic bundles (PU(2)-instantons). The main idea is to remark that (at least for small b_2) the absence of curves on a class VII surface X has a surprising consequence on the geometry of a certain instanton moduli space on X: the appearance of smooth compact connected component, whose points correspond to stable bundles. We show that the existence of such a component leads to a contradiction in the case $b_2 = 1$; we state the analogous result (existence of a cycle) in the case $b_2 = 2$, and we give the outline of the proof.

Abtract of (3) and (4). In these talks we give detailed proofs of crucial results stated and used in the previous conferences. For instance, we show that being able to write the "canonical extension"

$$0 \to \mathcal{K} \to \mathcal{A} \to \mathcal{O} \to 0$$

as an extension in a different way, implies the existence of a cycle. We also prove that a moduli space of simple bundles over a surface X with algebraic dimension $a(X) = 0$ cannot contain a Riemann surface which has both filtrable and non-filtrable points.