Report No. 2 の解答, 解説

Max 94. Average 56.

Q1.

(1) True.

Since ε and δ are positive infinitesimal, we have

$$0 < \varepsilon < a/2$$
$$0 < \delta < a/2$$

for arbitrary positive number a. Thus

$$0 < \varepsilon + \delta < a$$

This implies that $\varepsilon + \delta$ is positive infinitesimal.

(2) False

Let H be a positive infinite hyper-real number. If $\varepsilon = 1/H$, the product $\varepsilon \cdot H = 1$. This is finite.

If $\varepsilon = 1/H^2$, $\varepsilon \cdot H$ is positive infinitesimal.

(3) False.

Let H be a positive infinite hyper real number. If $K = \sqrt{H}$, then the quotient $H/K = \sqrt{H}$. This is infinite.

If $K = 1/H^2$, H/K is infinitesimal.

(4) True

Since ε is positive infinitesimal we have $0 < \varepsilon < a$ for arbitrary positive number a. On the other hand $H > b + a$ for arbitrary positive number b.

Thus $H - \varepsilon > b$. This implies $H - \varepsilon$ is positive infinite.

採点基準

(1) ~ (4) 真, 偽があっていれば +3

証明, 反例が正しければ +3

計 24 点.

Q2.

First, we consider the derivative of the function x^a, where a is a non negative integer. By definition

$$\frac{dx^a}{dx} = st\left(\frac{(x+\Delta x)^a - x^a}{\Delta x}\right),$$

where Δx is infinitesimal.

Since

$$(x+\Delta x)^a - x^a = ax^{a-1}\Delta x + \sum_{k=2}^{a} {}_kC_a x^{a-k}\Delta x^k,$$

we have

$$\frac{dx^a}{dx} = st\left(ax^{a-1} + \sum_{k=2}^{a} {}_kC_a x^{a-k}\Delta x^{k-1}\right)$$

$$= ax^{a-1}.$$

Next we consider the derivative of x^a, where $a < 0$. By the above argument,

$$\frac{dx^{-a}}{dx} = -ax^{-a-1}.$$

We consider the derivative of the both hand sides of the following equation:

$$x^a \cdot x^{-a} = 1.$$

By the rule of product

$$\frac{dx^a}{dx} \cdot x^{-a} + x^a \cdot (-a)x^{-a-1} = 0.$$

Thus

$$\frac{dx^a}{dx} = ax^{a-1}.$$

Thirdly, we consider the derivative of x^a, where $a = 1/n$ and n is a positive integer.

In this case, $y^n = x$. Since

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}},$$

we have

$$\frac{dy}{dx} = \frac{1}{ny^{n-1}} = \frac{1}{n \cdot x^{\frac{n-1}{n}}} = \frac{1}{n}x^{\frac{1}{n}-1}$$

$$= ax^{a-1}.$$

Fourthly, we consider the case x^a, where a is a rational number. Since a is rational, we have

$$a = \frac{m}{n}$$

where m and n are coprime integers.

Let $z = y^m$ and $y = x^{\frac{1}{n}}$. By the rule of chain,

$$\frac{dx^a}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx},$$

By the previous argument, we have

$$\frac{dz}{dy} = my^{m-1} \quad \text{and} \quad \frac{dy}{dx} = \frac{1}{n} x^{\frac{1}{n}-1}.$$

Thus

$$\frac{dx^a}{dx} = \frac{m}{n} x^{\left(\frac{m-1}{n} + \frac{1}{n} - 1\right)} = \frac{m}{n} x^{\frac{m}{n}-1}$$

$$= ax^{a-1} {}_{//}$$

採点基準.

通常の微分とTextでの微分の区別がついてない場合, 0点. x^a の a が整数の場合のみやってあるものは 10点を満点として適宜減点してある.

Q.3.

First we consider the derivative of $f(x)$ when x is an integer. By definition

$$f(x+\Delta x) = 0$$

for arbitrary infinitesimal numbers. Thus

$$f'(x) = st\left(\frac{f'(x+\Delta x) - f(x)}{\Delta x}\right)$$

$$= st\left(\frac{-1}{\Delta x}\right).$$

This implies that $f'(x)$ is undefined.

Next we consider the derivative of $f(x)$ when x is not integer. By definition

$$f(x+\Delta x) = 0$$

for arbitrary infinitesimal numbers. Thus

$$f'(x) = st\left(\frac{f(x+\Delta x) - f(x)}{\Delta x}\right) = st\left(\frac{0}{\Delta x}\right) = 0$$

Hence

$$f'(x) = \begin{cases} \text{undefined} & \text{if } x \text{ is an integer} \\ 0 & \text{other.} \end{cases}$$

採点基準.

上の結果が出ていれば10点. Textの定義に沿った説明がなされていれば+15点.

Q4.

Step 1 x … number of tickets.

p … price of each tickets

F … Total amount of incoming

Step 2. By the assumption,

$$0 \leqq x \leqq 60000,$$

$$p = 10 - x/10000 \quad \text{and}$$

$$F = px.$$

Step 3.

We consider the maximum of the function

$$y = 10x - x^2/10000.$$

when $0 \leqq x \leqq 60000.$

Since

$$\frac{dy}{dx} = 10 - \frac{x}{5000}$$

$$\frac{d^2y}{dx^2} = 10, \quad \text{and}$$

$$0 \leqq x \leqq 50000,$$

We find that y is maximal when $x = 50000.$ In that case, $p = 5.$

Ans. $x = 50000,\ p = 5.$

採点基準. 良く出来ていた. pを求めるのを忘れていたり, xの範囲に注意を払わなかったものは減点してある.