Quantitative homogenization in nonlinear elasiticity
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We consider a nonlinear elastic composite with a periodic micro-structure described by the non-convex integral functional

$$I_\varepsilon(u) = \int_\Omega W\left(\frac{x}{\varepsilon}, \nabla u(x)\right) - f(x) \cdot u(x)\, dx,$$

where $u : \Omega \to \mathbb{R}^d$ is the deformation, $f : \Omega \to \mathbb{R}^d$ is an external force, $\varepsilon > 0$ denotes the size of the micro-structure, and $W(y, F)$ is a stored energy function which is periodic in y. As it is well-known, under suitable growth conditions, I_ε Γ-converges to a functional with a homogenized energy density $W_{\text{hom}}(F)$, which is given by an *infinite-cell formula*. Under appropriate assumptions on W (namely, $p \geq d$-growth from below, frame indifference, minimality at identity, non-degeneracy and smoothness in a neighborhood close to the set of rotations) and on the microstructure, we show that in a neighbourhood of rotations the homogenized stored energy function W_{hom} is of class C^2 and characterized by a *single-cell homogenization formula*. Moreover, for small data, we establish an estimate on the homogenization error, which measures the distance between (almost) minimizers u_ε of I_ε and the minimizer of the homogenized problem. More precisely, we prove that the L^2-error as well as the H^1-error of the associated two-scale expansion decays with the rate $\sqrt{\varepsilon}$.

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