- Maslov's dequantization
- ultradiscretization

$R_\hbar$ $(\hbar>0)$ family of semirings $\mathbb{R}$ with

$$x +_\hbar y := \hbar \log\left(e^{\frac{x}{\hbar}} + e^{\frac{y}{\hbar}}\right), \quad x \times_\hbar y := x + y$$

$$\lim_{\hbar \to +0} \hbar \log\left(e^{\frac{x}{\hbar}} + e^{\frac{y}{\hbar}}\right) = \max\{x, y\}$$

$$R_0 = \mathbb{R}_{trop}$$

$R_\hbar \cong \mathbb{R}_{>0}$ (ordinary $+, \times$)
$(\hbar > 0)$

Non-Archimedean amoebas
(Kapranov)

K: the field of real power Puiseux series.

$$K \ni a = \sum_{j \in I_a} a_j t^j \qquad a_j \in \mathbb{C}^*$$

$I_a \subset \mathbb{R}$ well-ordered subset

$$\mathrm{val}(a) := -\min I_a \in \mathbb{R}$$

$$\mathrm{val} : K^* \to \mathbb{R}$$

$$\mathrm{val}(ab) = \mathrm{val}(a) + \mathrm{val}(b), \quad \mathrm{val}(a+b) \leq \max\{v(a), v(b)\}$$

$$\begin{tikzcd} \mathrm{Val} : (K^*)^n \arrow[r] \arrow[dr] & \mathbb{R}^n \\ & (\mathbb{C}^*)^n \arrow[u, "\mathrm{Log}"'] \end{tikzcd}$$

$$\mathrm{Val}(a_1, \cdots, a_n) := (\mathrm{val}(a_1), \cdots, \mathrm{val}(a_n))$$

$$\left(K^* \to \mathbb{C}^* \quad a \mapsto e^{\mathrm{val}(a) + \sqrt{-1}\arg(a_{-\mathrm{val}(a)})} \right)$$

$$f(z) = \sum_j a_j z^j \in K[z_1, z_2, \cdots, z_n]$$

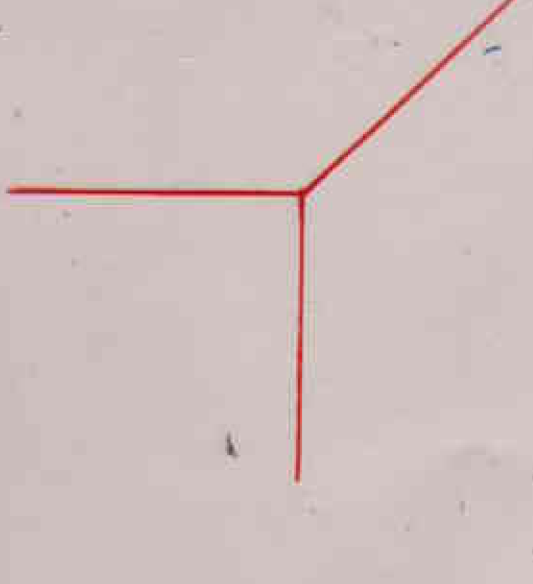
$$V_f := \{f(z) = 0\} \subset (K^*)^n$$

$$f^{trop}(x) := \text{"}\sum_j val(a_j) x^j\text{"}$$

$\underline{\underline{Th}}$ $\mathcal{A}_K := Val(V_f) = V_{f^{trop}}$

Example $f = z_1 + z_2 + 1 \in K[z_1, z_2]$

$$f_{trop} = \text{"}0x_1 + 0x_2 + 0\text{"}$$
$$= \max\{x_1, x_2, 0\}$$



Gromov-Witten invariant

$g \geqq 0,\ \Delta \subset \mathbb{R}^2$ fix

$s = \#(\partial\Delta \cap \mathbb{Z}^2)$

$N^{irr}(g,\Delta) := \#$ irreducible complex curves of genus g, degree Δ passing through a generic $s+g-1$-points in $(\mathbb{C}^*)^2$.

Th (Mikhalkin)

$$N^{irr}(g,\Delta) = N^{irr}_{trop}(g,\Delta)$$

$:= \#$ irr. tropical curves of genus g degree Δ passing through a generic $s+g-1$-points in $\mathbb{R}^2$

first Betti number

Degeneration of complex structures and complex tropical curves

$\hbar > 0 \quad \Phi_\hbar : (\mathbb{C}^*)^2 \longrightarrow (\mathbb{C}^*)^2$

$$(z_1, z_2) \longmapsto \left(|z_1|^{\hbar} \frac{z_1}{|z_1|}, |z_2|^{\hbar} \frac{z_2}{|z_2|} \right)$$

$$J_\hbar = (\Phi_\hbar)_*^{-1} \circ J \circ (\Phi_\hbar)_* : T(\mathbb{C}^*)^2 \to T(\mathbb{C}^*)^2$$

family of complex structures

Complex tropical curve

$J_\hbar$-hol. curves $\exists V_\hbar \longrightarrow V \quad (\hbar \to +0)$

(Hausdorff metric)

Th V: complex tropical curve

$\Longleftrightarrow \exists V_K \subset (K^*)^2$ alg. curve $/K$

$V = W(V_K)$

$$\begin{tikzcd} (K^*)^2 \arrow[rr, "W"] \arrow[dr, "\mathrm{Val}"'] & & (\mathbb{C}^*)^2 \arrow[dl, "\mathrm{Log}"] \\ & \mathbb{R}^2 & \end{tikzcd}$$

Real algebraic curves and Amoebas

$V \subset (\mathbb{C}^*)^n$ curve defined $/\mathbb{R}$

$V = V_f \quad f \in \mathbb{R}[z^{\pm 1}]$

- $\mathrm{Log}|_V : V \to \mathbb{R}^n$

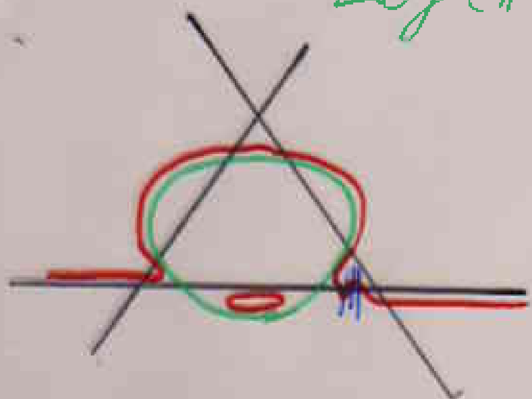
  $(\mathrm{Log} : (\mathbb{C}^*)^n \to \mathbb{R}^n \quad z \mapsto (\log|z_1|, \cdots$

$\mathbb{R}V \subseteq \mathrm{Crit}(\mathrm{Log}|V)$

V: simple Harnack curve

$\Rightarrow \mathrm{Log}|_{\mathbb{R}V} : \mathbb{R}V \to \mathbb{R}^n$ embedding

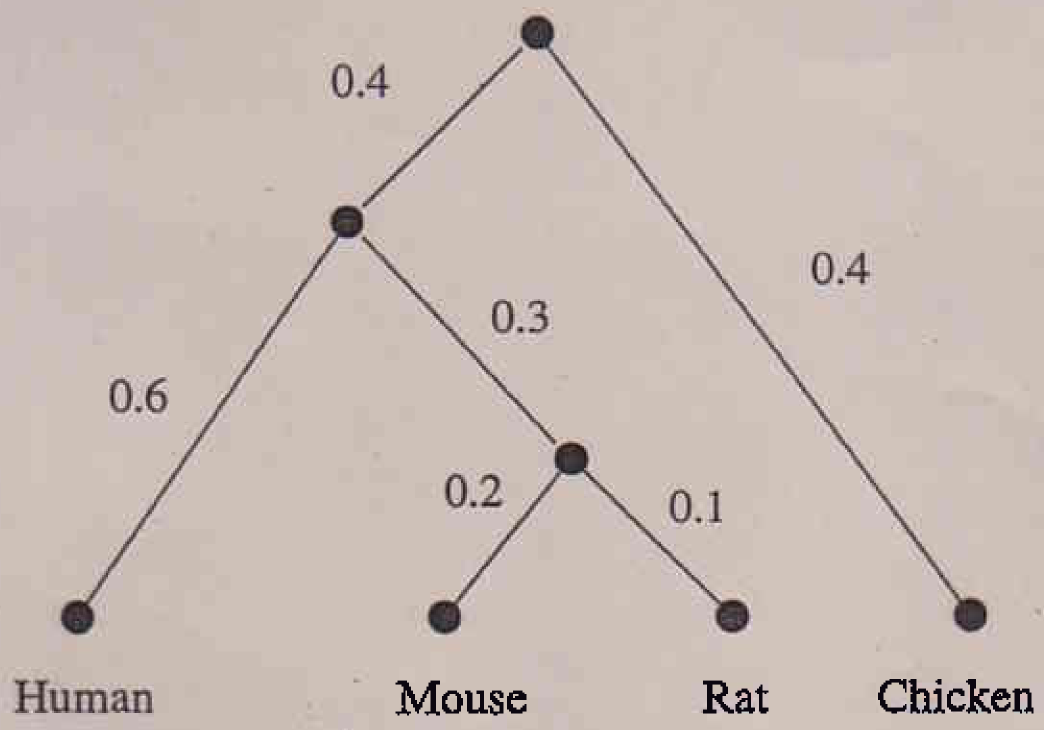
$\mathrm{Log}(\mathbb{R}V) = \partial\mathcal{A}$



Computational Biology

Human:	$ACAATGTCATTAGCGAT\ldots$
Mouse:	$ACGTTGTCAATAGAGAT\ldots$
Rat:	$ACGTAGTCATTACACAT\ldots$
Chicken:	$GCACAGTCAGTAGAGCT\ldots$

	H	M	R	C
H	0	1.1	1.0	1.4
M	1.1	0	0.3	1.3
R	1.0	0.2	0	1.2
C	1.4	1.3	1.2	0



A Phylogenetic Tree

Symmetric matrix

$$D = \begin{pmatrix} 0 & d_{12} & d_{13} & \cdots \\ d_{12} & 0 & d_{23} & \cdots \\ d_{13} & d_{23} & 0 & \\ \vdots & \vdots & & \ddots \end{pmatrix}$$

$\exists$ realizing polylogenetic tree

$\Rightarrow$ $D \cdot D = D$

$\uparrow$

min-plus semi-ring