質量ゼロの粒子をめぐって

大貫 義郎

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1 S-Matrix

$$\phi^{\text{out}} = S^\dagger \phi^{\text{in}} S, \qquad S^\dagger S = SS^\dagger = \mathbf{1} \tag{1.1}$$

ϕ^{in}, ϕ^{out}; Asymptotic Fields, (describing *Free* Particles.)

(World of Special Relativity)

$$|\,\text{out}, \alpha\rangle \equiv S|\,\text{in}, \alpha\rangle \tag{1.2}$$

$$S_{\alpha\beta} = \langle \text{in}, \alpha\,|\,\text{out}, \beta\rangle = \langle \text{in}, \alpha\,|S|\,\text{in}, \beta\rangle \tag{1.3}$$

$\{\,|\,\text{in}, \alpha\rangle\,\} \;\Leftrightarrow$ Fock Space

(Note: $c = \hbar = 1$, Pauli metric; e.g. $T_{\cdots 4 \cdots} = \mathrm{i}T_{\cdots 0 \cdots}$)

2 Massless Particles ($\boldsymbol{k}^2 = k_0^2$)

One-Body Amplitude $\phi_S^{(\pm)}(\boldsymbol{k}) \in$ Unitary Irrep. of Poincaré Group

$$S = 0,\ \pm 1/2,\ \pm 1,\ \pm 3/2,\ \cdots$$

$$S \gtrless 0 \quad \sim \quad \begin{matrix}\text{right}\\ \text{left}\end{matrix} \ \text{polarization}$$

$$(\pm) \quad \sim \quad k_0 \gtrless 0 \quad \sim \quad \begin{matrix}\text{positive}\\ \text{negative}\end{matrix} \ \text{frequency}$$

i.e.,

$$k_0 \phi_S^{(\pm)}(\boldsymbol{k}) = \pm|\boldsymbol{k}|\phi_S^{(\pm)}(\boldsymbol{k}) \tag{2.1}$$

Inner Product within an Irrep Space:

$$\langle \chi|\phi\rangle = \int \frac{d^3\boldsymbol{k}}{|\boldsymbol{k}|}\chi^*(\boldsymbol{k})\phi(\boldsymbol{k}) \tag{2.2}$$

1. Displacement: (b_μ)

$$x'_\mu = x_\mu - b_\mu$$

$$\phi'^{(\pm)}_S(\boldsymbol{k}) = e^{\mathrm{i}(\boldsymbol{k}\boldsymbol{b} \mp |\boldsymbol{k}| b_0)} \phi^{(\pm)}_S(\boldsymbol{k}) \tag{2.3}$$

2. (Infinitesimal) Space Rotation: $(\boldsymbol{\theta} = (\theta_1,\, \theta_2,\, \theta_3))$

$$\boldsymbol{x}' = \boldsymbol{x} + \boldsymbol{x} \times \boldsymbol{\theta}, \qquad x'_0 = x_0$$

$$\phi'^{(\pm)}_S(\boldsymbol{k}) = (1 + \mathrm{i}\boldsymbol{J} \cdot \boldsymbol{\theta})\, \phi^{(\pm)}_S(\boldsymbol{k}) \tag{2.4}$$

where

$$\begin{cases} J_1 = \dfrac{1}{\mathrm{i}}\Big(\boldsymbol{k} \times \dfrac{\partial}{\partial \boldsymbol{k}}\Big)_1 + \dfrac{k_1}{|\boldsymbol{k}| + k_3} S \\ J_2 = \dfrac{1}{\mathrm{i}}\Big(\boldsymbol{k} \times \dfrac{\partial}{\partial \boldsymbol{k}}\Big)_2 + \dfrac{k_2}{|\boldsymbol{k}| + k_3} S \\ J_3 = \dfrac{1}{\mathrm{i}}\Big(\boldsymbol{k} \times \dfrac{\partial}{\partial \boldsymbol{k}}\Big)_3 + S \end{cases}$$

Note:

$$\boldsymbol{J} \cdot \frac{\boldsymbol{k}}{|\boldsymbol{k}|} = S \tag{2.5}$$

3. (Infinitesimal) Lorentz Boost: ($\boldsymbol{\tau} = (\tau_1,\, \tau_2,\, \tau_3)$)

$$\boldsymbol{x}' = \boldsymbol{x} - \boldsymbol{\tau} x_0, \qquad x_0' = x_0 - \boldsymbol{x} \cdot \boldsymbol{\tau}$$

$$\phi'^{(\pm)}_S(\boldsymbol{k}) = (1 + \mathrm{i}\boldsymbol{K} \cdot \boldsymbol{\tau})\, \phi^{(\pm)}_S(\boldsymbol{k}) \tag{2.6}$$

$$\begin{cases} K_1 = -k_0\Big(\dfrac{1}{\mathrm{i}}\dfrac{\partial}{\partial k_1} - \dfrac{k_2}{|\boldsymbol{k}|(|\boldsymbol{k}| + k_3)} S\Big) \\ K_2 = -k_0\Big(\dfrac{1}{\mathrm{i}}\dfrac{\partial}{\partial k_2} + \dfrac{k_1}{|\boldsymbol{k}|(|\boldsymbol{k}| + k_3)} S\Big) \\ K_3 = -\dfrac{1}{\mathrm{i}} k_0 \dfrac{\partial}{\partial k_3} \end{cases}$$

$$k_0 \phi^{(\pm)}_S(\boldsymbol{k}) = \pm|\boldsymbol{k}| \phi^{(\pm)}_S(\boldsymbol{k}) \tag{2.7}$$

3 Covariant Amplitudes for $n \equiv 2|S| \geq 1$

Case for $|S| = 1/2$

$$\varphi^{R(+)}(\boldsymbol{k}) = \begin{pmatrix} \phi^{(+)}_{1/2}(\boldsymbol{k}) \\ 0 \end{pmatrix}, \qquad \varphi^{R(-)}(\boldsymbol{k}) = \begin{pmatrix} 0 \\ \phi^{(-)}_{-1/2}(\boldsymbol{k}) \end{pmatrix}$$

$$\varphi^{L(+)}(\boldsymbol{k}) = \begin{pmatrix} 0 \\ \phi^{(+)}_{-1/2}(\boldsymbol{k}) \end{pmatrix}, \qquad \varphi^{L(-)}(\boldsymbol{k}) = \begin{pmatrix} \phi^{(-)}_{1/2}(\boldsymbol{k}) \\ 0 \end{pmatrix}$$

$$\varphi^{R}(\boldsymbol{k}) = \varphi^{R(+)}(\boldsymbol{k}) + \varphi^{R(-)}(\boldsymbol{k})$$

$$\varphi^{L}(\boldsymbol{k}) = \varphi^{L(+)}(\boldsymbol{k}) + \varphi^{L(-)}(\boldsymbol{k})$$

$$k_0\varphi^R(\boldsymbol{k}) = |\boldsymbol{k}|\sigma_3\varphi^R(\boldsymbol{k}), \qquad k_0\varphi^L(\boldsymbol{k}) = -|\boldsymbol{k}|\sigma_3\varphi^L(\boldsymbol{k}) \tag{3.1}$$

Using the unitary matrix

$$U(\boldsymbol{k}) = \frac{1}{\sqrt{2|\boldsymbol{k}|(|\boldsymbol{k}| + k_3)}}\{|\boldsymbol{k}| + k_3 + \mathrm{i}(\sigma_1 k_2 - \sigma_2 k_1)\} \tag{3.2}$$

we define

$$\psi^{R/L}(\boldsymbol{k}) = \sqrt{|\boldsymbol{k}|}\, U(\boldsymbol{k})\varphi^{R/L}(\boldsymbol{k}). \tag{3.3}$$

Then

$$k_0\psi^{R/L}(\boldsymbol{k}) = (+/-)\boldsymbol{k}\boldsymbol{\sigma}\psi^{R/L}(\boldsymbol{k}) \tag{3.4}$$

and

1. Displacemen:

$$\psi'^{\,R/L}(\boldsymbol{k}) = e^{\mathrm{i}k_\mu b_\mu}\,\psi^{R/L}(\boldsymbol{k}) \tag{3.5}$$

2. Space Rotation:

$$\psi'^{\,R/L}(\boldsymbol{k}) = \Big(1 + \frac{\mathrm{i}}{2}\boldsymbol{\theta}\cdot\boldsymbol{\sigma}\Big)\psi^{R/L}(\boldsymbol{k} - \boldsymbol{k}\times\boldsymbol{\theta}) \tag{3.6}$$

3. Lorentz Boost:

$$\psi'^{\,R/L}(\boldsymbol{k}) = \Big(1 + (-/+)\frac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{\sigma}\Big)\psi^{R/L}(\boldsymbol{k} + \boldsymbol{\tau}k_0) \tag{3.7}$$

Case for $|\boldsymbol{S}| = \boldsymbol{n/2}$

$$\rho_1,\ \rho_2,\ \cdots,\ \rho_n = 1,\ 2$$

$$\varphi^{R(+)}_{(\rho_1\cdots\rho_n)}(\boldsymbol{k}) = \begin{cases} \phi^{(+)}_{n/2}(\boldsymbol{k}) & (\rho_1 = \rho_2 = \cdots = \rho_n = 1) \\ \ 0 & (\text{ otherwise}) \end{cases} \tag{3.8}$$

$$\varphi^{R(-)}_{(\rho_1\cdots\rho_n)}(\boldsymbol{k}) = \begin{cases} \phi^{(-)}_{-n/2}(\boldsymbol{k}) & (\rho_1 = \rho_2 = \cdots = \rho_n = 2) \\ \ 0 & (\text{ otherwise}) \end{cases} \tag{3.9}$$

$$\varphi^{L(+)}_{(\rho_1\cdots\rho_n)}(\boldsymbol{k}) = \begin{cases} \phi^{(+)}_{-n/2}(\boldsymbol{k}) & (\rho_1 = \rho_2 = \cdots = \rho_n = 2) \\ \ 0 & (\text{ otherwise}) \end{cases} \tag{3.10}$$

$$\varphi^{L(-)}_{(\rho_1\cdots\rho_n)}(\boldsymbol{k}) = \begin{cases} \phi^{(-)}_{n/2}(\boldsymbol{k}) & (\rho_1 = \rho_2 = \cdots = \rho_n = 1) \\ \ 0 & (\text{ otherwise}) \end{cases} \tag{3.11}$$

$$\varphi^{R/L}_{(\cdots)_n}(\boldsymbol{k}) = \varphi^{R/L(+)}_{(\cdots)_n}(\boldsymbol{k}) + \varphi^{R/L(-)}_{(\cdots)_n}(\boldsymbol{k}) \tag{3.12}$$

$$\Updownarrow$$

$$k_0\varphi^{R/L}_{(\cdots)_n}(\boldsymbol{k}) = (+/-)|\boldsymbol{k}|\sigma^{(i)}_3\varphi^{R/L}_{(\cdots)_n}(\boldsymbol{k}), \qquad (i = 1,\ 2,\ \cdots,\ n) \tag{3.13}$$

Then for

$$\psi^{R/L}_{(\cdots)_n}(\boldsymbol{k}) = |\boldsymbol{k}|^{n/2}\prod_{i=1}^{n} U^{(i)}(\boldsymbol{k})\varphi^{R/L}_{(\cdots)_n}(\boldsymbol{k}) \tag{3.14}$$

we obtain

$$k_0\psi^{R/L}_{(\cdots)_n}(\boldsymbol{k}) = (+/-)\boldsymbol{k}\boldsymbol{\sigma}^{(i)}\psi^{R/L}_{(\cdots)_n}(\boldsymbol{k}), \qquad (i = 1,\, 2,\, \cdots,\, n) \tag{3.15}$$

together with the followings:

1. Displacement

$$\psi'^{R/L}_{(\cdots)_n}(\boldsymbol{k}) = e^{\mathrm{i}k_\mu b_\mu}\psi^{R/L}_{(\cdots)_n}(\boldsymbol{k}) \tag{3.16}$$

2. Space Rotation

$$\psi'^{R/L}_{(\cdots)_n}(\boldsymbol{k}) = \Big(1 + \frac{\mathrm{i}}{2}\,\boldsymbol{\theta}\cdot\sum_{i=1}^{n}\boldsymbol{\sigma}^{(i)}\Big)\psi^{R/L}_{(\cdots)_n}(\boldsymbol{k} - \boldsymbol{k}\times\boldsymbol{\theta}) \tag{3.17}$$

3. Lorentz Boost

$$\psi'^{R/L}_{(\cdots)_n}(\boldsymbol{k}) = \Big(1 + (-/+)\frac{1}{2}\,\boldsymbol{\tau}\cdot\sum_{i=1}^{n}\boldsymbol{\sigma}^{(i)}\Big)\psi^{R/L}_{(\cdots)_n}(\boldsymbol{k} + \boldsymbol{\tau}k_0) \tag{3.18}$$

x-Representation:

$$\psi^{R/L}_{(\cdots)_n}(x) = \frac{1}{(2\pi)^{3/2}}\int \frac{d\boldsymbol{k}}{\sqrt{2}|\boldsymbol{k}|}\, e^{\mathrm{i}k_\mu x_\mu}\psi^{R/L}_{(\cdots)_n}(\boldsymbol{k})S \tag{3.19}$$

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Equations of Motion:

$$\Big(\frac{\partial}{\partial t} + (+/-)\boldsymbol{\sigma}^{(i)}\boldsymbol{\nabla}\Big)\psi^{R/L}_{(\cdots)_n}(x) = 0, \qquad (i = 1,\, 2,\, \cdots,\, n), \tag{3.20}$$

or equivalently

$$\gamma^{(i)}_\mu \partial_\mu \Big[\prod_{j=1}^{n} \frac{1}{2}(1 + (-/+)\gamma_5)^{(j)}\Big]\psi^{R/L}_{(\cdots)_n}(x) = 0, \qquad (i = 1,\, 2,\, \cdots,\, n)$$

where

$$\{\gamma_\mu,\, \gamma_\nu\} = 2\delta_{\mu\nu}, \qquad \gamma_5 \equiv \gamma_1\gamma_2\gamma_3\gamma_4.$$

Note:

$$\psi^{R/L}_{(\cdots)_n}(x) \iff \psi^{R/L}_{(\cdots)_n}(\boldsymbol{k}) \iff \varphi^{R/L}_{(\cdots)_n}(\boldsymbol{k}) \iff \phi^{(\pm)}_{n/2}(\boldsymbol{k}),\ \phi^{(\pm)}_{-n/2}(\boldsymbol{k}) \tag{3.21}$$

Example: (case for $n = 2$)

$$\psi^{R/L}_{(\rho_1\rho_2)}(x) \iff \begin{cases} F^R_{[\mu\nu]}(x) = F_{[\mu\nu]}(x) + \dfrac{1}{2}\epsilon_{\mu\nu\lambda\rho}F_{[\lambda\rho]}(x), \\ F^L_{[\mu\nu]}(x) = F_{[\mu\nu]}(x) - \dfrac{1}{2}\epsilon_{\mu\nu\lambda\rho}F_{[\lambda\rho]}(x) \end{cases} \tag{3.22}$$

$$\partial_\mu F^{R/L}_{[\mu\nu]}(x) = 0 \tag{3.23}$$

4 Quantized Fields

$$\phi^{(\pm)}_{n/2}(\boldsymbol{k}),\ \phi^{(\pm)}_{-n/2}(\boldsymbol{k}) \quad \Longrightarrow \quad \hat{\phi}^{(\pm)}_{n/2}(\boldsymbol{k}),\ \hat{\phi}^{(\pm)}_{-n/2}(\boldsymbol{k}) \tag{4.1}$$

Poincaré Invariant Nonvanishing Commutators:

$$[\hat{\phi}^{(+)}_{\pm n/2}(\boldsymbol{k}),\ \hat{\phi}^{(+)\dagger}_{\pm n/2}(\boldsymbol{k}')]_{\epsilon_n} = [\hat{\phi}^{(-)\dagger}_{\pm n/2}(\boldsymbol{k}),\ \hat{\phi}^{(-)}_{\pm n/2}(\boldsymbol{k}')]_{\epsilon_n} = |\boldsymbol{k}|\delta^3(\boldsymbol{k}-\boldsymbol{k}'), \tag{4.2}$$

$$\epsilon = (-)^{n+1}$$

$$\Downarrow$$

$$\begin{cases} [\hat{\psi}^R_{(\rho_1\rho_2\cdots\rho_n)}(x),\ \hat{\psi}^R_{(\rho'_1\rho'_2\cdots\rho'_n)}(x')]_{\epsilon_n} = 0, \\ [\hat{\psi}^R_{(\rho_1\rho_2\cdots\rho_n)}(x),\ \hat{\psi}^{R\dagger}_{(\rho'_1\rho'_2\cdots\rho'_n)}(x')]_{\epsilon_n} \\ \qquad = \dfrac{\mathrm{i}^{n+1}}{n!}\displaystyle\sum_{\tau\in S_n}\prod_{j=1}^{n}\big[\partial_t - (\boldsymbol{\sigma}^{(j)}\boldsymbol{\nabla})\big]_{\rho_j\,\rho'_{\tau j}} D(x-x') \end{cases} \tag{4.3}$$

and

$$\begin{cases} [\hat{\psi}^L_{(\rho_1\rho_2\cdots\rho_n)}(x),\ \hat{\psi}^L_{(\rho'_1\rho'_2\cdots\rho'_n)}(x')]_{\epsilon_n} = 0, \\ [\hat{\psi}^L_{(\rho_1\rho_2\cdots\rho_n)}(x),\ \hat{\psi}^{L\dagger}_{(\rho'_1\rho'_2\cdots\rho'_n)}(x')]_{\epsilon_n} \\ \qquad = \dfrac{\mathrm{i}^{n+1}}{n!}\displaystyle\sum_{\tau\in S_n}\prod_{j=1}^{n}\big[\partial_t + (\boldsymbol{\sigma}^{(j)}\boldsymbol{\nabla})\big]_{\rho_j\,\rho'_{\tau j}} D(x-x') \end{cases} \tag{4.4}$$

where

$$D(x) = -\frac{\mathrm{i}}{(2\pi)^3}\int d^4k\, e^{\mathrm{i}kx}\varepsilon(k_0)\delta(\boldsymbol{k}^2 - k_0^2). \tag{4.5}$$

(vanishing for space-like x)

5 Local Observables:

$$\hat{Q} = \int d^3\boldsymbol{x}\,\hat{Q}(x) + \text{const} \tag{5.1}$$

$$[\hat{Q}(x),\, \hat{Q}'(x')] = 0 \qquad \text{(for spacelike } (x - x')) \tag{5.2}$$

Especially, for massless free particles

$$\hat{Q}(x);\ \left\{ \begin{array}{l} \text{gauge-independent} \\ \\ \text{bilinear in } \partial^{\eta}\hat{\psi}^{R/L}_{(\cdots)_n}(x) \text{ and } \partial^{\eta'}\hat{\psi}^{R/L\,\dagger}_{(\cdots)_n}(x) \end{array} \right. \tag{5.3}$$

Hence

$$\hat{Q}(x) \sim \sum_{(\rho)_n,\,(\rho')_n,\,\eta+\eta'=N} c^{\eta\eta'}_{(\rho)_n,\,(\rho')_n}\,\partial^{\eta}\hat{\psi}^{R/L\,\dagger}_{(\rho)_n}(x)\,\partial^{\eta'}\hat{\psi}^{R/L}_{(\rho')_n}(x), \tag{5.4}$$

where

$$\begin{aligned} &c^{\eta\eta'}_{(\rho)_n,\,(\rho')_n}:\ \text{dimensionless consts.}, \\ &(\rho)_n = (\rho_1, \rho_2,\, \cdots,\, \rho_n), \\ &(\rho')_n = (\rho'_1, \rho'_2,\, \cdots,\, \rho'_n). \end{aligned}$$

Note:

$$[\,D(x)\,] = \mathrm{L}^{-2} \qquad \Rightarrow \qquad [\,\hat{\psi}^{R/L}_{(\rho)_n}(x)\,] = \mathrm{L}^{-(n/2+1)} \tag{5.5}$$

Thus

$$[\,\hat{Q}(x)\,] = \mathrm{L}^{-(n+2+N)} = \mathrm{L}^{-(2|S|+2+N)} \tag{5.6}$$

1. Energy-Momentum Density $\hat{P}^S_\mu(x)$

$$\hat{P}_\mu = \int d^3\boldsymbol{x}\, \hat{P}^S_\mu(x)$$
$$= \int \frac{d^3\boldsymbol{k}}{|\boldsymbol{k}|}\ k_\mu \big(\hat{\phi}^{(+)\dagger}_S(\boldsymbol{k})\hat{\phi}^{(+)}_S(\boldsymbol{k}) - \hat{\phi}^{(-)}_S(\boldsymbol{k})\hat{\phi}^{(-)\dagger}_S(\boldsymbol{k})\big), \tag{5.7}$$

$$[\hat{P}^S_\mu(x)] = \mathrm{L}^{-4} \quad \Rightarrow \quad |S| = 1 - \frac{N}{2}. \tag{5.8}$$

Cases $|S| \geq 3/2$ are not allowed.

2. Noether Charge Density $\hat{\rho}^S(x)$

Similarly,

$$\hat{\rho}^S = \int d^3\boldsymbol{x}\, \hat{\rho}^S(x)$$
$$= \int \frac{d^3\boldsymbol{k}}{|\boldsymbol{k}|}\ \big(\hat{\phi}^{(+)\dagger}_S(\boldsymbol{k})\hat{\phi}^{(+)}_S(\boldsymbol{k}) - \hat{\phi}^{(-)}_S(\boldsymbol{k})\hat{\phi}^{(-)\dagger}_S(\boldsymbol{k})\big). \tag{5.9}$$

$$[\hat{\rho}^S(x)] = \mathrm{L}^{-3} \quad \Rightarrow \quad |S| = \frac{(1-N)}{2}. \tag{5.10}$$

Cases $|S| \geq 1$ are not allowed.

$\Downarrow$

GRAVITON (neutral, massless, spin 2): $\sim$ NON-EXISTENT ? !

$\Downarrow$

Then what happens !

$\Downarrow$

Basic Assumptions

S-Matrix Theory ↓ Existence of Asymptotic World (AW) Described by Free Particles ↓ Free Quantum Fields ↓ Local Observables in AW

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A plausible answer to the problem would be CONFINEMENT OF GRAVITONS

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Related Topics:

S. Weinberg and E. Witten, Phys. Lett. **B96**, 59 (1980).

T. Kugo and S. Uehara, Prog. Theor. Phys. **66**, 1044 (1981).

S. Loebbert, Ann. Phys. (Berlin) **17**, 803 (2008).