A simple solution for one of the cosmological constant problems

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March 6, 2016

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Abstract

We propose a simple and totally covariant model which may solve one of the problems in the cosmological constant. The model is a kind of topological field theories and the contributions to the vacuum energy from the quantum corrections from the matters are absorbed into a redefinition of one of the scalar fields and the quantum corrections become irrelevant to the dynamics.

Problems in cosmological constant

Fine-tuning Problem, Coincidence Problem
(Definitions could not have been unified and different by authors.)

♠ Quantum correction
Observerd magnitude of the cosmological constant $\left(10^{-3}\,\mathrm{eV}\right)^4$ (very small).
$\Leftrightarrow$ Quantum corrections from the matter (vacuum energy) ρ_{vacuum} diverge.
$\Rightarrow$ Cutoff scale $\Lambda_{\mathrm{cutoff}}$ to regularize the divergence:

$$\rho_{\mathrm{vacuum}} = \frac{1}{(2\pi)^3}\int d^3k \frac{1}{2}\sqrt{k^2+m^2} \sim \Lambda_{\mathrm{cutoff}}^4$$

$\cdots$ much larger than the observed value $\left(10^{-3}\,\mathrm{eV}\right)^4$ of the energy density in the universe.

If the supersymmetry is restored in the high energy,

$$\rho_{\mathrm{vacuum}} = \frac{1}{(2\pi)^3}\int d^3k \frac{1}{2}\left(\sqrt{k^2 + m_{\mathrm{boson}}^2} - \sqrt{k^2 + m_{\mathrm{fermion}}^2}\right)$$
$$\sim \Lambda_{\mathrm{cutoff}}^2 \Lambda_{\not{\mathrm{SUSY}}}^2$$

$\Lambda_{\not{\mathrm{SUSY}}}$: scale of supersymmetry breaking.
Assumed $\Lambda_{\not{\mathrm{SUSY}}}^2 = m_{\mathrm{boson}}^2 - m_{\mathrm{fermion}}^2$.

Anyway the vacuum energy coming from the quantum corrections is very large and if we use the counter term in order to obtain the very small vacuum energy $\left(10^{-3}\,\mathrm{eV}\right)^4$, we need very very fine-tuning and extremely unnatural.

Sequestering models

- N. Kaloper and A. Padilla, "Sequestering the Standard Model Vacuum Energy," Phys. Rev. Lett. **112** (2014) 9, 091304 [arXiv:1309.6562 [hep-th]].
- N. Kaloper and A. Padilla, "Vacuum Energy Sequestering: The Framework and Its Cosmological Consequences," Phys. Rev. D **90** (2014) 8, 084023 [Phys. Rev. D **90** (2014) 10, 109901] [arXiv:1406.0711 [hep-th]].
- N. Kaloper, A. Padilla, D. Stefanyszyn and G. Zahariade, "A Manifestly Local Theory of Vacuum Energy Sequestering," arXiv:1505.01492 [hep-th].

Unimodular gravity (much older)

- J. L. Anderson and D. Finkelstein, "Cosmological constant and fundamental length," Am. J. Phys. **39** (1971) 901. doi:10.1119/1.1986321
- W. Buchmuller and N. Dragon, "Einstein Gravity From Restricted Coordinate Invariance," Phys. Lett. B **207** (1988) 292. doi:10.1016/0370-2693(88)90577-1
- M. Henneaux and C. Teitelboim, "The Cosmological Constant and General Covariance," Phys. Lett. B **222** (1989) 195. doi:10.1016/0370-2693(89)91251-3
- W. G. Unruh, "A Unimodular Theory of Canonical Quantum Gravity," Phys. Rev. D **40** (1989) 1048. doi:10.1103/PhysRevD.40.1048
- Y. J. Ng and H. van Dam, "Unimodular Theory of Gravity and the Cosmological Constant," J. Math. Phys. **32** (1991) 1337. doi:10.1063/1.529283

etc.

Unimodular constraint

$$\sqrt{-g} = 1$$

$\Rightarrow g^{\mu\nu}\delta g_{\mu\nu} = 0 \Rightarrow$ Cosmological constant is irrelevant
Lose full invariance under reparametrization.

Lagrangian formalism constraint $\Leftarrow$ Lagrange multiplier field λ

$$S = \int d^4x \left\{ \sqrt{-g} \left(\mathcal{L}_{\rm gravity} - \lambda \right) + \lambda \right\} + S_{\rm matter}$$

$S_{\rm matter}$: action of matters,
$\mathcal{L}_{\rm gravity}$: Lagrangian density of arbitrary gravity models.

Divide the gravity Lagrangian density

$$\mathcal{L}_{\rm gravity} = \mathcal{L}^{(0)}_{\rm gravity} - \Lambda$$

Redefine λ: $\lambda \to \lambda - \Lambda \Rightarrow$

$$S = \int d^4 x \left\{ \sqrt{-g} \left(\mathcal{L}^{(0)}_{\rm gravity} - \lambda \right) + \lambda \right\} + S_{\rm matter} + \Lambda \int d^4 x \, .$$

Last term $\Lambda \int d^4 x$ does not depend on any dynamical variable

$\Rightarrow$ may drop the last term.

Obtained action does not include the cosmological constant.

Cosmological constant Λ does not affect the dynamics even in the action.

Cosmological constant may include the large quantum corrections from matters to the vacuum energy.

$\Rightarrow$ Large quantum corrections can be tuned to vanish.

New model (totally covariant)

$$S = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{\rm gravity} - \lambda \left(1 - \frac{1}{\mu^4} \nabla_\mu J^\mu \right) \right\} + S_{\rm matter}$$

μ: a constant with a mass dimension,
J^μ: a general vector quantity,
∇_μ: ia covariant derivative

$\mathcal{L}_{\rm gravity} = \mathcal{L}^{(0)}_{\rm gravity} - \Lambda$, $\lambda \to \lambda \to \lambda - \Lambda \Rightarrow$

$$S = \int d^4x \sqrt{-g} \left\{ \mathcal{L}^{(0)}_{\rm gravity} - \lambda \left(1 - \frac{1}{\mu^4} \nabla_\mu J^\mu \right) \right\} + S_{\rm matter} - \frac{\Lambda}{\mu^4} \int d^4x \sqrt{-g} \nabla_\mu J^\mu$$

The integrand in the last term is total derivative
$\Rightarrow$ the last term does not affect any dynamics and we may drop the last term, again.

We may choose $\nabla_\mu J^\mu$ to be a topological invariant like

- Gauss-Bonnet invariant

$$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

- For abelian gauge theory,

$$I \equiv \frac{1}{4!}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

- Instanton density for non-abelian abelian gauge theory,

$$I \equiv \frac{1}{4!}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\, F_{\mu\nu}F_{\rho\sigma}$$

Simplest model

$J^\mu \propto \partial^\mu \varphi$:

$$S = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{\rm gravity} - \lambda \left(1 + \frac{1}{\mu^3} \nabla_\mu \partial^\mu \varphi \right) \right\} + S_{\rm matter}$$
$$= \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{\rm gravity} - \lambda + \frac{1}{\mu^3} \partial_\mu \lambda \partial^\mu \varphi \right\} + S_{\rm matter}$$

The term $\frac{1}{\mu^3}\partial_\mu \lambda \partial^\mu \varphi \Rightarrow$ this model may include a ghost.

Redefinitionof scalar fields φ and λ,

$$\varphi = \frac{1}{\sqrt{2}} \left(\eta + \xi \right) \, , \quad \lambda = \frac{\mu^3}{\sqrt{2}} \left(\eta - \xi \right) \, ,$$

$\Rightarrow$

$$S = \int d^4x\sqrt{-g}\left\{\mathcal{L}_{\rm gravity} - \frac{1}{2}\partial_\mu\xi\partial^\mu\xi + \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \frac{\mu^3}{\sqrt{2}}\left(\eta - \xi\right)\right\} + S_{\rm matter}$$

η generates the negative norm state and therefore η is a ghost.
$\Rightarrow$ Introducing the fermionic (Grassmann odd) ghosts b and c,

$$S' = \int d^4x\sqrt{-g}\left\{\mathcal{L}_{\rm gravity} - \lambda + \frac{1}{\mu^3}\partial_\mu\lambda\partial^\mu\varphi - \partial_\mu b\partial^\mu c\right\} + S_{\rm matter}\,.$$

Invariant under BRS transformation

$$\delta\lambda = \delta c = 0\,, \quad \delta\varphi = \epsilon c\,, \quad \delta b = \frac{1}{\mu^3}\epsilon\lambda$$

ϵ: fermionic parameter.

Defining the physical states as the states invariant under the BRS transformation, the negative norm states can be consistently removed.

- T. Kugo and I. Ojima, "Manifestly Covariant Canonical Formulation of Yang-Mills Field Theories: Physical State Subsidiary Conditions and Physical S Matrix Unitarity," Phys. Lett. B **73** (1978) 459. doi:10.1016/0370-2693(78)90765-7
- T. Kugo and I. Ojima, "Local Covariant Operator Formalism of Nonabelian Gauge Theories and Quark Confinement Problem," Prog. Theor. Phys. Suppl. **66** (1979) 1. doi:10.1143/PTPS.66.1

Conserved ghost number, 1 for c and -1 for b and ϵ
(λ, φ, b, c): a quartet in Kugo-Ojima's quartet mechanism.

Topological Field Theory

Lagrangian density,

$$\mathcal{L} = -\lambda + \frac{1}{\mu^3}\partial_\mu \lambda \partial^\mu \varphi - \partial_\mu b \partial^\mu c$$

regarded as the Lagrangian density of a topological field theory

- E. Witten, "Topological Quantum Field Theory," Commun. Math. Phys. **117** (1988) 353. doi:10.1007/BF01223371

Lagrangian density is BRS exact, that is, given by the BRS transformation of some quantity.

Start with field theory of φ but $\mathcal{L}_\varphi = 0$
$\Rightarrow$ Under any transformation of φ, the Lagrangian density is trivially invariant
$\Rightarrow$ gauge theory.

Gauge condition,

$$1 + \frac{1}{\mu^3} \nabla_\mu \partial^\mu \varphi = 0$$

Gauge-fixing Lagrangian + ghost Lagrangian = BRS transformation of $-b\left(1 + \frac{1}{\mu^3}\nabla_\mu \partial^\mu \varphi\right)$.

$$\begin{aligned}
&\delta\left(-b\left(1 + \frac{1}{\mu^3}\nabla_\mu \partial^\mu \varphi\right)\right) \\
&= \epsilon\left(-\lambda\left(1 + \frac{1}{\mu^3}\nabla_\mu \partial^\mu \varphi\right) + b\nabla_\mu \partial^\mu c\right) \\
&= \epsilon\left(\mathcal{L} + (\text{total derivative terms})\right)
\end{aligned}$$

- T. Kugo and S. Uehara, Nucl. Phys. B **197** (1982) 378. doi:10.1016/0550-3213(82)90449-7

Lagrangian density: BRS exact up to total derivative.

Cosmology

Cosmplogical evolution

$$\mathcal{L}_{\rm gravity} = \frac{R}{2\kappa^2} - \Lambda$$

(Einstein gravity)
R: scalar curvature, κ: gravitational coupling constant.

FRW metric with flat spacial part,

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^{3} \left(dx^i\right)^2 \ ,$$

Assume λ, φ only depend on t. $a(t)$: scale factor.

Eq. for φ

$$0 = 1 + \frac{1}{\mu^3}\left(\frac{d^2\varphi}{dt^2} + 3H\frac{d\varphi}{dt}\right)$$

Hubble rate $H \equiv \frac{1}{a}\frac{da}{dt}$.

Eq. for λ

$$0 = \frac{d^2\lambda}{dt^2} + 3H\frac{d\lambda}{dt}$$

Neglect contributions from matters.
(t,t) and (i,j) components of Einstein equation,

$$\begin{aligned}\frac{3}{\kappa^2}H^2 =& \Lambda + \lambda - \frac{d\lambda}{dt}\frac{d\varphi}{dt}\,,\\ -\frac{1}{\kappa^2}\left(3H^2 + 2\frac{dH}{dt}\right) =& -\Lambda - \lambda - \frac{d\lambda}{dt}\frac{d\varphi}{dt}\end{aligned}$$

Deleting Λ,

$$\frac{1}{\kappa^2}\frac{dH}{dt} = \frac{d\lambda}{dt}\frac{d\varphi}{dt} \,.$$

A solution for λ: $\lambda = \lambda_0$ (constant) $\Rightarrow H = H_0$ (constant)

$$\Rightarrow \quad \lambda_0 = -\Lambda + \frac{3H_0^2}{\kappa^2} \,, \quad \varphi = -\frac{t}{3H_0}$$

$H = H_0 \Rightarrow$ de Sitter space-time but H_0 does not depend on Λ.
H_0 could be determined by initial condition or something else.
Value of Λ is irrelevant for the cosmology.

Summary

- We have proposed a simple and totally covariant model, which is a topologcal field theory and may solve the problem of the quantum corrections from the matters to the vacuum energy.
- The mechanism is similar to that in the unimodular gravity but the variation of the scalar field λ does not give any constraint on the metric like the unimoduar constraint, but the variation of λ gives the equation for another scalar field φ.