重力子が質量をもつ重力理論と二つの計量をもつ重力理論

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Preliminary

"bigravity" = system of massive spin 2 field (massive graviton)
+ gravity (includes massless spin 2 field = graviton)

Fierz-Pauli action (linearized or free theory), 3/4 century ago

M. Fierz and W. Pauli, "On relativistic wave equations for particles of arbitrary spin in an electromagnetic field," Proc. Roy. Soc. Lond. A **173** (1939) 211.

The Lagrangian of the massless spin-two field (graviton) $h_{\mu\nu}$ is given by

$$\mathcal{L}_0 = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\lambda h^\lambda{}_\mu \partial_\nu h^{\mu\nu} - \partial^\mu h_{\mu\nu}\partial^\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h \,, \quad \left(h \equiv h^\mu{}_\mu\right) \,.$$

Massless graviton: 2 degrees of freedom (helicity),
Massive graviton: 5 degrees of freedom ($2s+1$, spin $s=2$).

The Lagrangian of the massive graviton with mass m is given by

$$\mathcal{L}_m = \mathcal{L}_0 - \frac{m^2}{2}\left(h_{\mu\nu}h^{\mu\nu} - h^2\right) \quad \text{(Fierz-Pauli action)} .$$

When $m = 0$, gauge symmetry (linearized general covariance)

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu ,$$

$\xi_\mu(x)$: space-time dependent gauge parameter.
The combination $h_{\mu\nu}h^{\mu\nu} - h^2$:

Fierz-Pauli tuning (not related with any symmetry)

For the combination $h_{\mu\nu}h^{\mu\nu} - (1-a)h^2$,

if $a \neq 0$, there appears ghost scalar field with mass

$$m_g^2 = \frac{3-4a}{2a} m^2 \quad \left(m_g^2 \to \infty \text{ when } a \to 0\right)$$

Hamiltonian and counting of degrees of freedom:

$\frac{D(D-1)}{2} - 1$ propagating degrees of freedom in D dimensions (5 degrees of freedom for $D = 4$).

Legendre transformation only with respect to the spatial components h_{ij}.

$$\pi_{ij} = \frac{\partial \mathcal{L}}{\partial \dot{h}_{ij}} = \dot{h}_{ij} - \dot{h}_{kk}\delta_{ij} - 2\partial_{(i}h_{j)0} + 2\partial_k h_{0k}\delta_{ij} ,$$

$$\Rightarrow \; S = \int d^D x \left\{ \pi_{ij}\dot{h}_{ij} - \mathcal{H} + 2h_{0i}\left(\partial_j \pi_{ij}\right) + m^2 h_{0i}^2 \right.$$

$$\left. + h_{00}\left(\vec{\nabla}^2 h_{ii} - \partial_i\partial_j h_{ij} - m^2 h_{ii}\right) \right\} ,$$

$$\mathcal{H} = \frac{1}{2}\pi_{ij}^2 - \frac{1}{2}\frac{1}{D-2}\pi_{ii}^2 + \frac{1}{2}\partial_k h_{ij}\partial_k h_{ij} - \partial_i h_{jk}\partial_j h_{ik}$$

$$+ \partial_i h_{ij}\partial_j h_{kk} - \frac{1}{2}\partial_i h_{jj}\partial_i h_{kk} + \frac{1}{2}m^2(h_{ij}h_{ij} - h_{ii}^2) .$$

$m = 0$ case: h_{0i}, h_{00}: Lagrange multipliers $\rightarrow$ constraints

$$\partial_j \pi_{ij} = 0\,, \quad \vec{\nabla}^2 h_{ii} - \partial_i \partial_j h_{ij} = 0\,.$$

First class constraints $\rightarrow$ gauge symmetry ($\Leftarrow$ general covariance)

For $D = 4$, h_{ij} and π_{ij} each have 6 components, respectively.
$\rightarrow$ 12 dimensional phase space.
4 constraints $+$ 4 gauge invariances
$\rightarrow$ 4 dimensional phase space
(two polarizations (helicities) of massless graviton)

$m \neq 0$: h_{0i} are no longer Lagrange multipliers $\delta h_{0i} \;\Rightarrow\; h_{0i} = -\frac{1}{m^2}\partial_j \pi_{ij}$,

$$S = \int d^D x \left\{ \pi_{ij}\dot{h}_{ij} - \mathcal{H} + h_{00}\left(\vec{\nabla}^2 h_{ii} - \partial_i \partial_j h_{ij} - m^2 h_{ii} \right) \right\} ,$$

$$\mathcal{H} = \frac{1}{2}\pi_{ij}^2 - \frac{1}{2}\frac{1}{D-2}\pi_{ii}^2 + \frac{1}{2}\partial_k h_{ij}\partial_k h_{ij} - \partial_i h_{jk}\partial_j h_{ik}$$
$$+ \partial_i h_{ij}\partial_j h_{kk} - \frac{1}{2}\partial_i h_{jj}\partial_i h_{kk} + \frac{1}{2}m^2\left(h_{ij}h_{ij} - h_{ii}^2 \right) + \frac{1}{m^2}\left(\partial_j \pi_{ij}\right)^2 .$$

h_{00}: Lagrange multiplier $\to$ single constraint

$$\mathcal{C} = -\vec{\nabla}^2 h_{ii} + \partial_i \partial_j h_{ij} + m^2 h_{ii} = 0 ,$$

Secondary constraint:

$$\{H, \mathcal{C}\}_{\mathrm{PB}} = \frac{1}{D-2} m^2 \pi_{ii} + \partial_i \partial_j \pi_{ij} = 0 , \quad H = \int d^d x \; \mathcal{H} ,$$

Two second class constraints.

For $D = 4$,
12 dimensional phase space $-$ 2 constraints $= 10$ degrees of freedom
(5 polarizations of the massive graviton and their conjugate momenta).

vDVZ(van Dam, Veltman, and Zakharov) discontinuity

H. van Dam and M. J. G. Veltman, "Massive and massless Yang-Mills and gravitational fields," Nucl. Phys. B **22** (1970) 397.

V. I. Zakharov, "Linearized gravitation theory and the graviton mass," JETP Lett. **12** (1970) 312 [Pisma Zh. Eksp. Teor. Fiz. **12** (1970) 447].

Discontinuity of $m \to 0$ limit in the free massive gravity with the Einstein gravity due to the extra degrees of freedom in the limit.
$\Rightarrow$ the Vainstein mechanism

A. I. Vainshtein, "To the problem of nonvanishing gravitation mass," Phys. Lett. B **39** (1972) 393.

Non-linearity screens the extra degrees of freedom (non-linearity becomes strong when m is small).

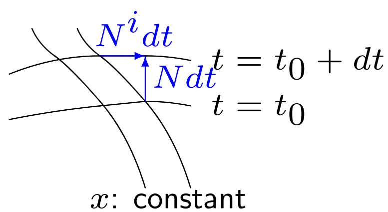
Boulware-Deser ghost

D. G. Boulware and S. Deser, "Classical General Relativity Derived from Quantum Gravity," Annals Phys. **89** (1975) 193.

In non-linear (interacting) theory, 6th degree of freedom appears as a ghost.

Non-linear massive gravity action with flat metric $\eta_{\mu\nu}$, $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$

$$S = \frac{1}{2\kappa^2}\int d^D x \left[\sqrt{-g}R - \frac{1}{4}m^2 \eta^{\mu\alpha}\eta^{\nu\beta}\left(h_{\mu\nu}h_{\alpha\beta} - h_{\mu\alpha}h_{\nu\beta}\right)\right] .$$



ADM formalism (N: lapse function, N_i: shift function)

$$g_{00} = -N^2 + g^{ij}N_i N_j , \quad g_{0i} = N_i , \quad g_{ij} = g_{ij} .$$

$i, j, \cdots = 1, 2, 3$, g^{ij}: inverse of the spatial metric g_{ij}.

$m = 0$ case

Einstein-Hilbert action (after partial integrations)

$$\frac{1}{2\kappa^2}\int d^D x\sqrt{g}N\left[{}^{(d)}R - K^2 + K^{ij}K_{ij}\right] \ ,$$

${}^{(d)}R$: curvature of spatial metric g_{ij}, K_{ij}: extrinsic curvature

$$K_{ij} = \frac{1}{2N}\left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right) \ ,$$

∇_i: covariant derivative w.r.t. the spatial metric g_{ij}.
Canonical momenta with respect to g_{ij}:

$$p^{ij} = \frac{\delta L}{\delta \dot{g}_{ij}} = \frac{1}{2\kappa^2}\sqrt{g}\left(K^{ij} - K g^{ij}\right) \ ,$$

Hamiltonian:

$$H = \left(\int_{\Sigma_t} d^d x \; p^{ab} \dot{g}_{ab} \right) - L = \int_{\Sigma_t} d^d x \; N\mathcal{C} + N_i \mathcal{C}^i \, .$$

$$\mathcal{C} = \sqrt{g} \left[{}^{(d)}R + K^2 - K^{ij} K_{ij} \right] \, , \quad \mathcal{C}^i = 2\sqrt{g} \nabla_j \left(K^{ij} - K h^{ij} \right) \, ,$$

$$K_{ij} = \frac{2\kappa^2}{\sqrt{g}} \left(p_{ij} - \frac{1}{D-2} p h_{ij} \right) \, .$$

For $m = 0$, Hamiltonian vanishes. N, N_i: Lagrange multipliers
$\Rightarrow \mathcal{C} = 0$, $\mathcal{C}_i = 0$: first class constraints $\Leftrightarrow$ general covariance
In $D = 4$,
12 phase space metric components $-$ 4 constraints $-$ 4 gauge symmetries
$= 4$ phase space degrees of freedom
$=$ degrees of freedom in linearized theory of massless spin 2 graviton

$m \neq 0$ case $(h_{ij} \equiv g_{ij} - \delta_{ij})$

$$\eta^{\mu\alpha}\eta^{\mu\beta}\left(h_{\mu\nu}h_{\alpha\beta} - h_{\mu\alpha}h_{\mu\beta}\right)$$
$$= \delta^{ik}\delta^{jl}\left(h_{ij}h_{kl} - h_{ik}h_{jl}\right) + 2\delta^{ij}h_{ij} - 2N^2\delta^{ij}h_{ij} + 2N_i\left(g^{ij} - \delta^{ij}\right)N_i\,,$$

Action

$$S = \frac{1}{2\kappa^2}\int d^Dx\Big\{p^{ab}\dot{g}_{ab} - N\mathcal{C} - N_i\mathcal{C}^i - \frac{m^2}{4}\left[\delta^{ik}\delta^{jl}\left(h_{ij}h_{kl} - h_{ik}h_{jl}\right) + 2\delta^{ij}h_{ij} - 2N^2\delta^{ij}h_{ij} + 2N_i\left(g^{ij} - \delta^{ij}\right)N_j\right]\Big\}\,.$$

N^2, N_iN_j terms $\Rightarrow$ N^2, N_i: *Not* Lagrange multipliers but auxiliary fields.

$$N = \frac{\mathcal{C}}{m^2 \delta^{ij} h_{ij}}, \quad N_i = \frac{1}{m^2} \left(g^{ij} - \delta^{ij} \right)^{-1} \mathcal{C}^j .$$

No constraints nor gauge symmetries.
Hamiltonian:

$$H = \frac{1}{2\kappa^2} \int d^d x \left\{ \frac{1}{2m^2} \frac{\mathcal{C}^2}{\delta^{ij} h_{ij}} + \frac{1}{2m^2} \mathcal{C}^i \left(g^{ij} - \delta^{ij} \right)^{-1} \mathcal{C}^j \right.$$
$$\left. + \frac{m^2}{4} \left[\delta^{ik} \delta^{jl} \left(h_{ij} h_{kl} - h_{ik} h_{jl} \right) + 2 \delta^{ij} h_{ij} \right] \right\} .$$

12 phase space degrees of freedom, or 6 real degrees of freedom.
One more degree of freedom, compared with linearized theory
$\Rightarrow$ ghost scalar

Boulware-Deser ghost

Massive gravity without ghost

C. de Rham and G. Gabadadze, “Generalization of the Fierz-Pauli Action,” Phys. Rev. D **82**, 044020 (2010) [arXiv:1007.0443 [hep-th]],

C. de Rham, G. Gabadadze and A. J. Tolley, “Resummation of Massive Gravity,” Phys. Rev. Lett. **106** (2011) 231101 [arXiv:1011.1232 [hep-th]].

S. F. Hassan and R. A. Rosen, “Resolving the Ghost Problem in non-Linear Massive Gravity,” Phys. Rev. Lett. **108** (2012) 041101 [arXiv:1106.3344 [hep-th]].

Non-dynamical metric $f_{\mu\nu}\,(\sim \eta_{\mu\nu})$, $\sqrt{g^{-1}f}$: $\sqrt{g^{-1}f}\sqrt{g^{-1}f} = g^{\mu\lambda}f_{\lambda\nu}$

Minimal extension of Fierz-Pauli action:

$$S = M_p^2 \int d^4x \sqrt{-g} \left[R - 2m^2 \left(\mathrm{tr}\,\sqrt{g^{-1}f} - 3 \right) \right] .$$

$\Rightarrow$ vDVZ discontinuity $\Rightarrow$

$$S = M_p^2 \int d^4x \sqrt{-g} \left[R + 2m^2 \sum_{n=0}^{3} \beta_n \, e_n(\sqrt{g^{-1}f}) \right] ,$$

$$e_0(\mathbb{X}) = 1 \,, \quad e_1(\mathbb{X}) = [\mathbb{X}] \,, \quad e_2(\mathbb{X}) = \tfrac{1}{2}([\mathbb{X}]^2 - [\mathbb{X}^2]) \,,$$

$$e_3(\mathbb{X}) = \tfrac{1}{6}([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3]) \,,$$

$$e_4(\mathbb{X}) = \tfrac{1}{24}([\mathbb{X}]^4 - 6[\mathbb{X}]^2[\mathbb{X}^2] + 3[\mathbb{X}^2]^2 + 8[\mathbb{X}][\mathbb{X}^3] - 6[\mathbb{X}^4]) \,,$$

$$e_k(\mathbb{X}) = 0 \quad \text{for } k > 4 \,,$$

$$\mathbb{X} = (X^\mu{}_\nu) \ , \quad [\mathbb{X}] \equiv X^\mu{}_\mu \,,$$

$\sim$ Galileon $\Rightarrow$ Vainshtein mechanism

(longitudinal scalar mode ($h_{\mu\nu} \sim \partial_\mu \partial_\nu \phi$) $\sim$ Galileon scalar field)

Hamiltonian constraint: Minimal extension case

ADM formulation, $f_{\mu\nu} = \eta_{\mu\nu} \Rightarrow$

$$\mathcal{L} = \pi^{ij}\partial_t \gamma_{ij} + N R^0 + N^i R_i - 2m^2 \sqrt{\gamma}\, N \left(\mathrm{tr}\sqrt{g^{-1}\eta} - 3 \right) .$$

$$(g^{-1}\eta)^{\mu}{}_{\nu} = \frac{1}{N^2} \begin{pmatrix} 1 & N^l \delta_{lj} \\ -N^i & (N^2\gamma^{il} - N^i N^l)\delta_{lj} \end{pmatrix} , \quad N^i = \gamma^{ij} N_j .$$

Highly nonlinear action in $N_\mu \Rightarrow$ New combinations n^i

$$N^i = (\delta^i_j + N D^i{}_j) n^j ,$$

$$D^i{}_j : \ (\sqrt{1 - n^T \mathbf{I}\, n})\, D = \sqrt{(\gamma^{-1} - D n n^T D^T)\mathbf{I}} ,$$

$$\mathbf{I} = \delta_{ij} , \quad \mathbf{I}^{-1} = \delta^{ij} ,$$

$$\Rightarrow \ \mathcal{L} = \pi^{ij}\partial_t\gamma_{ij} + NR^0 + R_i(\delta^i_j + ND^i{}_j)n^j$$
$$- 2m^2\sqrt{\gamma}\left[\sqrt{1 - n^T\mathbf{I}n} + N\mathrm{tr}\,(\sqrt{\gamma^{-1}\mathbf{I} - Dnn^TD^T\mathbf{I}}) - 3N\right].$$

Linear in N.

$\delta n_i \ \Rightarrow \ n^i = -R_j\delta^{ji}\left[4m^4\det\gamma + R_k\delta^{kl}R_l\right]^{-1/2}$: Not including N.

$$\delta N \ \Rightarrow R^0 + R_iD^i{}_jn^j - 2m^2\sqrt{\gamma}\left[\sqrt{1 - n^r\delta_{rs}n^s}\ D^k{}_k - 3\right] = 0\,.$$

$+$ secondary constraint $= 2$ constraints.

12 components of γ_{ij} and π^{ij} $-$ 2 constraints
$= 10$ components (massive spin 2)

Bimetric gravity (bigravity)

S. F. Hassan and R. A. Rosen, "Bimetric Gravity from Ghost-free Massive Gravity," JHEP **1202** (2012) 126 [arXiv:1109.3515 [hep-th]].

Dynamical $f_{\mu\nu}$ (background independent).

$$S = M_g^2 \int d^4x \sqrt{-\det g}\, R^{(g)} + M_f^2 \int d^4x \sqrt{-\det f}\, R^{(f)}$$

$$+ 2m^2 M_{\text{eff}}^2 \int d^4x \sqrt{-\det g} \sum_{n=0}^{4} \beta_n \, e_n(\sqrt{g^{-1}f}) \,,$$

$$1/M_{\text{eff}}^2 \equiv 1/M_g^2 + 1/M_f^2 \,.$$

$R^{(g)}$: scalar curvature for $g_{\mu\nu}$, $R^{(f)}$: scalar curvature for $f_{\mu\nu}$.

Spectrum of the linearized theory

Minimal case: $\beta_0 = 3$, $\beta_1 = -1$, $\beta_2 = 0$, $\beta_3 = 0$, $\quad \beta_4 = 1$.

Linearize $\quad g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{M_g} h_{\mu\nu}\,, \quad f_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{M_f} l_{\mu\nu}\,,$

$$\Rightarrow \; S = \int d^4x \, (h_{\mu\nu} \hat{\mathcal{E}}^{\mu\nu\alpha\beta} h_{\alpha\beta} + l_{\mu\nu} \hat{\mathcal{E}}^{\mu\nu\alpha\beta} l_{\alpha\beta})$$
$$- \frac{m^2 M_{\mathrm{eff}}^2}{4} \int d^4x \left[\left(\frac{h^\mu{}_\nu}{M_g} - \frac{l^\mu{}_\nu}{M_f} \right)^2 - \left(\frac{h^\mu{}_\mu}{M_g} - \frac{l^\mu{}_\mu}{M_f} \right)^2 \right] .$$

$\hat{\mathcal{E}}^{\mu\nu\alpha\beta}$: usual Einstein-Hilbert kinetic operator.

Change of variables

$$\frac{1}{M_{\rm eff}}\,u_{\mu\nu} = \frac{1}{M_f}\,h_{\mu\nu} + \frac{1}{M_g}\,l_{\mu\nu}\,, \qquad \frac{1}{M_{\rm eff}}\,v_{\mu\nu} = \frac{1}{M_g}\,h_{\mu\nu} - \frac{1}{M_f}\,l_{\mu\nu}\,.$$

$\Rightarrow$

$$S = \int d^4x\,(u_{\mu\nu}\hat{\mathcal{E}}^{\mu\nu\alpha\beta}u_{\alpha\beta} + v_{\mu\nu}\hat{\mathcal{E}}^{\mu\nu\alpha\beta}v_{\alpha\beta})$$
$$- \frac{m^2}{4}\int d^4x\,\left(v^{\mu\nu}v_{\mu\nu} - v^{\mu}{}_{\mu}v^{\nu}{}_{\nu}\right)\,.$$

One massless spin-2 particle $u_{\mu\nu}$ and one massive spin-2 particle $v_{\mu\nu}$ with mass m.

Renormalizable theory of massive spin two particle

Y. Ohara, S. Akagi and S. Nojiri, "Renormalizable theory of massive spin two particle and new bigravity," arXiv:1402.5737 [hep-th].

New ghost free interactions

K. Hinterbichler, "Ghost-Free Derivative Interactions for a Massive Graviton," JHEP **1310** (2013) 102 [arXiv:1305.7227 [hep-th]].

$$\begin{aligned}\mathcal{L}_{d,n} \sim& \eta^{\mu_1\nu_1\cdots\mu_n\nu_n}\partial_{\mu_1}\partial_{\nu_1}h_{\mu_2\nu_2}\cdots\partial_{\mu_{d-1}}\partial_{\nu_{d-1}}h_{\mu_d\nu_d}h_{\mu_{d+1}\nu_{d+1}}\cdots h_{\mu_{n+d/2}nu_{n+d/2}}\,,\\ h_{\mu\nu} =& g_{\mu\nu} - \eta_{\mu\nu}\,,\end{aligned}$$

"pseudo linear terms

$$\begin{aligned}\eta^{\mu_1\nu_1\mu_2\nu_2} \equiv& \eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2} - \eta^{\mu_1\nu_2}\eta^{\mu_2\nu_1}\,,\\ \eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} \equiv& \eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2}\eta^{\mu_3\nu_3} - \eta^{\mu_1\nu_1}\eta^{\mu_2\nu_3}\eta^{\mu_3\nu_2} + \eta^{\mu_1\nu_2}\eta^{\mu_2\nu_3}\eta^{\mu_3\nu_1}\\ &- \eta^{\mu_1\nu_2}\eta^{\mu_2\nu_1}\eta^{\mu_3\nu_3} + \eta^{\mu_1\nu_3}\eta^{\mu_2\nu_1}\eta^{\mu_3\nu_2} - \eta^{\mu_1\nu_3}\eta^{\mu_2\nu_2}\eta^{\mu_3\nu_1}\,.\end{aligned}$$

Linear with respect to h_{00} in the Hamiltonian.
Do not appear the terms which include both of h_{00} and h_{0i}.
Variation of h_{00} $\Rightarrow$ a constraint for h_{ij} and their conjugate momenta π_{ij}
$\Rightarrow$ eliminate the ghost.
Power-counting renormalizable model of the massive spin two particle

$$
\begin{aligned}
\mathcal{L}_{h0} =& \frac{1}{2}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\left(\partial_{\mu_1}\partial_{\nu_1}h_{\mu_2\nu_2}\right)h_{\mu_3\nu_3} - \frac{m^2}{2}\eta^{\mu_1\nu_1\mu_2\nu_2}h_{\mu_1\nu_1}h_{\mu_2\nu_2} \\
& - \frac{\mu}{3!}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3} - \frac{\lambda}{4!}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4} \\
=& \frac{1}{2}\left(h\Box h - h^{\mu\nu}\Box h_{\mu\nu} - h\partial^\mu\partial^\nu h_{\mu\nu} - h_{\mu\nu}\partial^\mu\partial^\nu h + 2h_\nu^{\ \rho}\partial^\mu\partial^\nu h_{\mu\rho}\right) \\
& - \frac{m^2}{2}\left(h^2 - h_{\mu\nu}h^{\mu\nu}\right) - \frac{\mu}{3!}\left(h^3 - 3hh_{\mu\nu}h^{\mu\nu} + 2h_\mu^{\ \nu}h_\nu^{\ \rho}h_\rho^{\ \mu}\right) \\
& - \frac{\lambda}{4!}\left(h^4 - 6h^2h_{\mu\nu}h^{\mu\nu} + 8hh_\mu^{\ \nu}h_\nu^{\ \rho}h_\rho^{\ \mu} - 6h_\mu^{\ \nu}h_\nu^{\ \rho}h_\rho^{\ \sigma}h_\sigma^{\ \mu} + 3\left(h_{\mu\nu}h^{\mu\nu}\right)^2\right) .
\end{aligned}
$$

m, μ: parameters with the dimension of mass
λ: dimensionless parameters.
$\Rightarrow$ power-counting renormalizable (free from ghost)

Propagator

$$D^m_{\alpha\beta,\rho\sigma} = \frac{1}{2\left(p^2+m^2\right)}\left\{P^m_{\alpha\rho}P^m_{\beta\sigma} + P^m_{\alpha\sigma}P^m_{\beta\rho} - \frac{2}{D-1}P^m_{\alpha\beta}P^m_{\rho\sigma}\right\},$$
$$P^m_{\mu\nu} \equiv \eta_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}.$$

$p^2 \to \infty \Rightarrow D^m_{\alpha\beta,\rho\sigma} \sim \mathcal{O}\left(p^2\right) \cdots$ Not be renormalizable

Similar problem in the model of massive vector field

$$\mathcal{L} = -\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) - \frac{1}{2}m^2 A^\mu A_\mu \,.$$

Propagator

$$D_{\mu\nu} = -\frac{1}{p^2 + m^2} P^m_{\mu\nu} \,,$$

which is the inverse of

$$O^{\mu\nu} \equiv -\left(p^2 + m^2\right)\eta^{\mu\nu} + p^\mu p^\nu \,, \quad O^{\mu\nu} D_{\nu\rho} = \delta^\mu{}_\rho \,.$$

$p^2 \to \infty \Rightarrow D_{\mu\nu} \sim \mathcal{O}(1)$: Not renormalizable

If the vector field, however, couples only with the conserved current J_μ $(\partial^\mu J_\mu = 0)$
$\Rightarrow \frac{p_\mu p_\nu}{m^2}$ in $P^m_{\mu\nu}$ drops
$\Rightarrow D_{\mu\nu} \sim \mathcal{O}\left(1/p^2\right) \Rightarrow$ maybe renormalizable.

Instead of imposing the conservation law, we may add the term $2\alpha\phi\partial^\mu A_\mu$
Inverse of the operator

$$O_{A\phi} = \begin{pmatrix} O^{\mu\nu} & -i\alpha p^\mu \\ i\alpha p^\nu & 0 \end{pmatrix},$$

is given by

$$D_{A\phi} = \begin{pmatrix} -\frac{1}{p^2+m^2} P_{\nu\rho} & -i\frac{p_\nu}{\alpha p^2} \\ i\frac{p_\rho}{\alpha p^2} & \frac{m^2}{\alpha^2 p^2} \end{pmatrix}, \quad P^{\mu\nu} \equiv \eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2},$$
$$\left(O_{A\phi} D_{A\phi} = \begin{pmatrix} \delta^\mu{}_\rho & 0 \\ 0 & 1 \end{pmatrix} \right).$$

$P_{\mu\nu} = P^m_{\mu\nu}$ on shell, $p^2 = -m^2$
$p^2 \to \infty \Rightarrow$ Propagator between twoA_μ's $\mathcal{O}\left(1/p^2\right)$
$\Rightarrow$ Renormalizable if the interaction terms are also renormalizable.

Adding the term $4\alpha A^{\mu}\partial^{\nu}h_{\mu\nu}$

$$\begin{pmatrix} \mathcal{O}^{\mu\nu,\alpha\beta} & -i\alpha\left(p^{\mu}\eta^{\alpha\nu}+p^{\nu}\eta^{\alpha\mu}\right) \\ i\alpha\left(p^{\alpha}\eta^{\mu\beta}+p^{\beta}\eta^{\mu\alpha}\right) & 0 \end{pmatrix}\begin{pmatrix} D_{\alpha\beta,\rho\sigma} & -iE_{\sigma,\alpha\beta} \\ iE_{\alpha,\rho\sigma} & F_{\alpha\sigma} \end{pmatrix}$$
$$=\begin{pmatrix} \frac{1}{2}\left(\delta^{\mu}{}_{\rho}\delta^{\nu}{}_{\sigma}+\delta^{\mu}{}_{\alpha}\delta^{\nu}{}_{\beta}\right) & 0 \\ 0 & \delta^{\mu}{}_{\sigma} \end{pmatrix}.$$

$\Rightarrow$

$$D_{\alpha\beta,\rho\sigma}=-\frac{1}{2\left(p^2+m^2\right)}\left\{P_{\alpha\rho}P_{\beta\sigma}+P_{\alpha\sigma}P_{\beta\rho}-\frac{2}{D-2}P_{\alpha\beta}P_{\rho\sigma}\right\},$$
$$E_{\alpha,\rho\sigma}=\frac{1}{2\alpha p^2}\left\{p_{\rho}P_{\alpha\sigma}+p_{\sigma}P_{\alpha\rho}-\frac{m^2p_{\alpha}}{(D-2)\left(p^2+m^2\right)}P_{\rho\sigma}+\frac{p_{\alpha}p_{\rho}p_{\sigma}}{p^2}\right\},$$
$$F_{\alpha\sigma}=\frac{m^2}{2\alpha^2p^2}P_{\alpha\sigma}+\frac{(D-1)\,m^4}{4\alpha^2(D-2)\left(p^2\right)^2\left(p^2+m^2\right)}p_{\alpha}p_{\sigma}.$$

Propagator between two $h_{\mu\nu}$'s $\mathcal{O}\left(1/p^2\right)\Rightarrow$ maybe renormalizable.

New bigravity

Couples with gravity $\sim$ new bigravity

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} \nabla_{\mu_1} \nabla_{\nu_1} h_{\mu_2\nu_2} h_{\mu_3\nu_3} - \frac{1}{2} m^2 g^{\mu_1\nu_1\mu_2\nu_2} h_{\mu_1\nu_1} h_{\mu_2\nu_2} \right.$$
$$- \frac{\mu}{3!} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} h_{\mu_1\nu_1} h_{\mu_2\nu_2} h_{\mu_3\nu_3} - \frac{\lambda}{4!} g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} h_{\mu_1\nu_1} h_{\mu_2\nu_2} h_{\mu_3\nu_3} h_{\mu_4\nu_4}$$
$$\left. + 4\alpha A^\mu \nabla^\nu h_{\mu\nu} \right\} ,$$

$h_{\mu\nu}$ is not the perturbation in $g_{\mu\nu}$ but $h_{\mu\nu}$ is a field independent of $g_{\mu\nu}$.
Cosmology with the Einstein-Hilbert action:

$$S_{\mathrm{EH}} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} R \, .$$

Assume $h_{\mu\nu} = C g_{\mu\nu}$, C: constant

$$S = -\int d^4x \sqrt{-g} V(C) \, , \quad V(C) \equiv 6m^2 C + 4\mu C^3 + \lambda C^4 \, , \quad (\nabla_\rho g_{\mu\nu} = 0) \ .$$

$C \Leftarrow V'(C) = 0$.
Parametrize m^2 and μ by

$$m^2 = \frac{\lambda}{3} C_1 C_2 \, , \quad \mu = -\frac{\lambda}{3} (C_1 + C_2) \ .$$

$\Rightarrow C = 0$, C_1, C_2

$$V(C_1) = \frac{\lambda}{3} C_1^3 (-C_1 + 2C_2) \ , \quad V(C_2) = \frac{\lambda}{3} C_2^3 (-C_2 + 2C_1) \ .$$

$V(C) \sim$ cosmological constant.

Summary

- Brief review on massive gravity and bigravity.
- Proposition of a renormalizable theory describing massive spin two particle.
- The coupling of the theory with gravity $\rightarrow$ a new kind of bimetric gravity or bigravity.
  - The field of the massive spin two particle plays the role of the cosmological constant.

$F(R)$ bigravity