

# Covariant power-counting renormalizable gravity with Lagrange multipliers

Based on the works with S.D. Odintsov,

Phys.Rev.D81:043001,2010, arXiv:0905.4213 [hep-th];

Phys.Lett.B691:60-64,2010, arXiv:1004.3613 [hep-th];

Phys.Rev.D83:023001,2011, arXiv:1007.4856 [hep-th]

and J. Kluson,

Phys.Lett.B701:117-126,2011, arXiv:1104.4286 [hep-th] .

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# Introduction

Petr Hořava, “Quantum Gravity at a Lifshitz Point”

Phys.Rev.D79:084008,2009, arXiv:0901.3775 [hep-th]

- A candidate of quantum field theory of gravity which is power-counting renormalizable.
- Anisotropy between space and time (explicit breaking of covariance).  
At long distances, the Lorentz symmetry could be recovered.

Problem: the existence of extra scalar mode violating the Newton law.

**Today's talk:** A Proposal of covariant and power-counting renormalizable model of gravity, where only massless graviton propagates.

# Brief review on Hořava gravity

Einstein's general relativity:

non-renormalizable as a quantum field theory

In the flat background:  $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$  ( $\kappa$ : gravitational coupling)

$$\begin{aligned} S &= \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} R \\ &= \int d^4x \left[ -\frac{1}{2} \partial h \partial h + \kappa h \partial h \partial h + \kappa^2 h^2 \partial h \partial h + \dots \right. \\ &\quad \left. + \kappa^n h^n \partial h \partial h + \dots \right] \end{aligned}$$

The dimension of  $\kappa$  :  $[L]$  ( $L$ : length)  $\Rightarrow$  **non-renormalizable**

If the propagator  $1/p^2 \rightarrow 1/p^4, 1/p^6, \dots$  ( $p_\mu$ : four momentum)  
 $\Rightarrow$  UV behavior could be improved.  $\Rightarrow$  higher derivative theory  
unitarity could be broken in general, due the higher derivative with  
respect to  $t$

Hořava's idea:

anisotropic treatment between space and time

higher derivative theory with respect to only spacial coordinates

( $\mathbf{p}$ : spacial momentum).

$$1/p^2 \rightarrow 1/\mathbf{p}^4, 1/\mathbf{p}^6, \dots, (|\mathbf{p}| \rightarrow \infty)$$

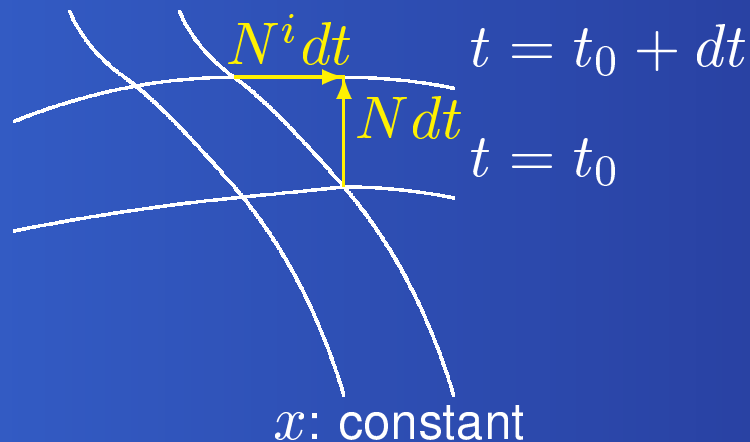
Anisotropy: Under the scale transformation with a constant  $b$ ,

$$\mathbf{x} \rightarrow b\mathbf{x}, t \rightarrow b^z t, (z = 2, 3, \dots)$$

# ADM decomposition

## Metric

$$ds^2 = -N^2 dt^2 + g_{ij}^{(3)} (dx^i + N^i dt) (dx^j + N^j dt) , \quad (i = 1, 2, 3)$$



$N$  : lapse variable

$N^i$  : shift variable

$$\int d^4x \sqrt{-g} R \Rightarrow \int d^3x dt N \sqrt{g^{(3)}} \left( K^{ij} K_{ij} - K^2 + R^{(3)} \right)$$

Extrinsic curvature :  $K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i) , \quad K = K^i_i$

## Action of Hořava gravity: kinetic term + “potential”

Kinetic term (including the derivatives with respect to time  $\sim (\partial_t)^2$ )

$$S_K = \frac{2}{\kappa^2} \int dt d^3\mathbf{x} \sqrt{g} N (K_{ij} K^{ij} - \lambda K^2), \quad \lambda : \text{parameter}$$

Symmetry: spacial diffeomorphism  $\otimes$  temporal diffeomorphism

$$\delta x^i = \zeta^i(t, \mathbf{x}), \quad \delta t = f(t)$$

Dimension of time  $t$  :  $[L^z]$  ( $L$ : length)

$$ds^2 = -N^2 dt^2 + \dots \Rightarrow [N] = [L^{1-z}]$$

In perturbation theory, gauge fixing :  $N = N_0$

Effective coupling constant:  $1/\kappa_{\text{eff}}^2 = N_0/\kappa^2$ ,  $[\kappa_{\text{eff}}^2] = [L^{3-z}]$

When  $z = 3$ ,  $\kappa_{\text{eff}}$ : dimensionless  $\Rightarrow$  **power counting renormalizable**

“Potential” (not including the derivatives with respect to time)

Generalized De Witt “metric on the space of metrics”

$$\mathcal{G}^{ijkl} = \frac{1}{2} \left( g^{ik} g^{j\ell} + g^{i\ell} g^{jk} \right) - \lambda g^{ij} g^{kl}$$

Potential with “detailed balance”

$$S_V = \frac{\kappa^2}{8} \int dt d^3 \mathbf{x} \sqrt{g} N E^{ij} \mathcal{G}_{ijkl} E^{kl} \quad \sqrt{g} E^{ij} = \frac{\delta W[g_{kl}]}{\delta g_{ij}}$$

$z = 2$  model  $\dots (\partial_t)^2 \leftrightarrow (\partial_x^2)^2$

$$W = \frac{1}{\kappa_W^2} \int d^3 \mathbf{x} \sqrt{g} (R - 2\Lambda_W)$$

$$\Rightarrow S_V = \frac{\kappa^2}{8\kappa_W^4} \int dt d^3 \mathbf{x} \sqrt{g} N \left( R^{ij} - \frac{1}{2} R g^{ij} + \Lambda_W g^{ij} \right) \mathcal{G}_{ijkl} \left( R^{kl} - \frac{1}{2} R g^{kl} + \Lambda_W g^{kl} \right)$$

$$z = 3 \text{ model} \cdots (\partial_t)^2 \leftrightarrow (\partial_x^3)^2$$

$$W = \frac{1}{w^2} \int_{\Sigma} \omega_3(\Gamma) \quad w^2 : \text{dimensionless coupling}$$

$\omega_3(\Gamma)$  : gravitational Chern-Simons term

$$\omega_3(\Gamma) = \text{Tr} \left( \Gamma \wedge d\Gamma + \frac{2}{3} \Gamma \wedge \Gamma \wedge \Gamma \right) \equiv \varepsilon^{ijk} \left( \Gamma_{il}^m \partial_j \Gamma_{km}^\ell + \frac{2}{3} \Gamma_{il}^n \Gamma_{jm}^\ell \Gamma_{kn}^m \right) d^3 \mathbf{x}$$

$\Rightarrow$

$$\begin{aligned} S &= \int dt d^3 \mathbf{x} \sqrt{g} N \left\{ \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) - \frac{\kappa^2}{2w^4} C_{ij} C^{ij} \right\} \\ &= \int dt d^3 \mathbf{x} \sqrt{g} N \left\{ \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) \right. \\ &\quad \left. - \frac{\kappa^2}{2w^4} \left( \nabla_i R_{jk} \nabla^i R^{jk} - \nabla_i R_{jk} \nabla^j R^{ik} - \frac{1}{8} \nabla_i R \nabla^i R \right) \right\} \end{aligned}$$

$$\text{Cotton tensor} : C^{ij} = \varepsilon^{ikl} \nabla_k \left( R_\ell^j - \frac{1}{4} R \delta_\ell^j \right)$$



## Problems on Hořava gravity

Symmetry: spacial diffeomorphism  $\otimes$  temporal diffeomorphism

$$\delta x^i = \zeta^i(t, \mathbf{x}), \quad \delta t = f(t)$$

**Projectability condition**: Assume  $N = N(t)$  from the beginning

Degrees of freedom of Hořava gravity  $\neq$  Degrees of freedom of Einstein gravity

$\Rightarrow$  does not reproduce general relativity even in the low energy region

C. Charmousis, G. Niz, A. Padilla, P.M. Saffin,

“Strong coupling in Horava gravity” arXiv:0905.2579

Miao Li, Yi Pang, “A Trouble with Hořava-Lifshitz Gravity”, arXiv:0905.2751

D. Blas, O. Pujolas, S. Sibiryakov,

“Consistent Extension of Horava Gravity” arXiv:0909.3525

etc.

# Covariant Model

Correct degrees of freedom

$\Leftrightarrow$  Covariance (full diffeomorphism invariance)

$\Rightarrow$  Proposal of covariant and renormalizable models:

S.N., S.D. Odintsov, + J. Kluson

arXiv:0905.4213 [hep-th]; arXiv:1004.3613 [hep-th]; arXiv:1007.4856 [hep-th];

arXiv:1104.4286 [hep-th]

c.f. P. Hořava, C.M. Melby-Thompson,

"General Covariance in Quantum Gravity at a Lifshitz Point",

Phys.Rev. D82 (2010) 064027, arXiv:1007.2410 [hep-th]

Idea:

Spontaneous breakdown of Lorentz symmetry:  $\partial_\mu \phi \neq 0$

The breakdown  $\Leftarrow$  constraint

$\sim$  Stückelberg formulation of massive vector field

## Stückelberg formulation of massive vector field

Lagrangian

$$\mathcal{L} = -\frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu) (\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{2} m^2 (\partial_\mu \varphi - A_\mu) (\partial^\mu \varphi - A^\mu)$$

Gauge symmetry

$$A_\mu \rightarrow A_\mu + \partial_\mu \epsilon, \quad \varphi \rightarrow \varphi + \epsilon$$

Gauge fixing condition:  $\varphi = 0 \Rightarrow$  massive vector field

$$\mathcal{L} = -\frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu) (\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{2} m^2 A_\mu A^\mu$$

the variation of  $\varphi \Rightarrow$  physical state condition

By using constrained complex scalar field  $\eta$  ( $\eta^* \eta = 1$ )

$$\mathcal{L} = -\frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu) (\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{2} m^2 \{(\partial_\mu - iA_\mu) \eta^*\} \{(\partial^\mu + iA^\mu) \eta\} \quad \eta = e^{i\varphi}$$

which is obtained from the double-well potential for complex scalar field  $\zeta$ :

$$V = \frac{1}{4} M^2 (\zeta^* \zeta - 1)^2 \quad \zeta = \rho e^{i\varphi}$$

In the limit of  $M \rightarrow \infty$  ( $\rho \rightarrow 1$ )

By replacing  $\eta$  : non-trivial representation of  $U(1)$

→ constrained vector field  $\partial_\mu \phi$  or  $A_\mu$

⇒ spontaneous breakdown of Lorentz symmetry

Models (two kinds of models):

$$\begin{aligned}
 S_{2n+2} = & \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} - \alpha \{ (\partial^\mu \phi \partial^\nu \phi \nabla_\mu \nabla_\nu - \partial_\mu \phi \partial^\mu \phi \nabla^\rho \nabla_\rho)^n \right. \\
 & \times P_\alpha^\mu P_\beta^\nu \left( R_{\mu\nu} - \frac{1}{2U_0} \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi \right) \Big\} \\
 & \times \left\{ (\partial^\mu \phi \partial^\nu \phi \nabla_\mu \nabla_\nu - \partial_\mu \phi \partial^\mu \phi \nabla^\rho \nabla_\rho)^n P^{\alpha\mu} P^{\beta\nu} \left( R_{\mu\nu} - \frac{1}{2U_0} \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi \right) \right\} \\
 & \left. - \lambda \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + U_0 \right) \right],
 \end{aligned}$$

for  $z = 2n + 2$  model ( $n = 0, 1, 2, \dots$ )

$\lambda$ : Lagrange multiplier field

$U_0$ : constant

$P_\mu^\nu$ : projection operator  $P_\mu^\nu \equiv \delta_\mu^\nu + \frac{\partial_\mu \phi \partial^\nu \phi}{2U_0}$

$$\begin{aligned}
S_{2n+3} = & \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} - \alpha \{ (\partial^\mu \phi \partial^\nu \phi \nabla_\mu \nabla_\nu - \partial_\mu \phi \partial^\mu \phi \nabla^\rho \nabla_\rho)^n \right. \\
& \times P_\alpha^\mu P_\beta^\nu \left( R_{\mu\nu} - \frac{1}{2U_0} \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi \right) \left. \right\} \\
& \times \left\{ (\partial^\mu \phi \partial^\nu \phi \nabla_\mu \nabla_\nu - \partial_\mu \phi \partial^\mu \phi \nabla^\rho \nabla_\rho)^{n+1} P^{\alpha\mu} P^{\beta\nu} \left( R_{\mu\nu} - \frac{1}{2U_0} \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi \right) \right\} \\
& \left. - \lambda \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + U_0 \right) \right].
\end{aligned}$$

for  $z = 2n + 3$  model ( $n = 0, 1, 2, \dots$ )

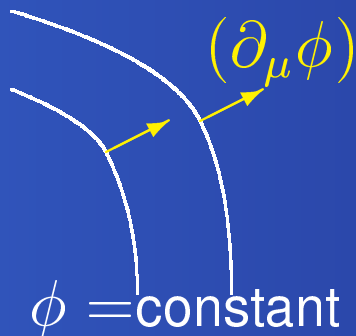
$\lambda$ : Lagrange multiplier field

$$\Rightarrow \text{constraint} : \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + U_0 = 0 \Rightarrow (\partial_\mu \phi) : \text{time-like}$$

Locally, one can choose the direction of time to be parallel to  $(\partial_\mu \phi)$

$$\frac{1}{2} \left( \frac{d\phi}{dt} \right)^2 = U_0$$

Spacial region :  $\phi = \text{constant}$  hypersurface (equipotential surface)



Spontaneous breakdown of  
Lorentz symmetry

The actions admit a flat space vacuum solution.

Field equations:

$$0 = \frac{1}{2\kappa^2} \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) + G_{\mu\nu}^{\text{higher}} - \frac{\lambda}{2} \partial_\mu \phi \partial_\nu \phi + \frac{\lambda}{2} g_{\mu\nu} \left( \frac{1}{2} \partial_\rho \phi \partial^\rho \phi + U_0 \right) .$$

Assuming the flat vacuum solution,

$$0 = \lambda \partial_\mu \phi \partial_\nu \phi .$$

$$\Rightarrow \lambda = 0 .$$

$\Rightarrow$  Flat space vacuum solution with  $\lambda = 0$ .



## Perturbation from the flat background

Diffeomorphism invariance with respect to time coordinate

$$\Rightarrow \phi = \sqrt{2U_0}t \quad (\text{unitary}) \text{ gauge condition .}$$

Perturbation from flat background:  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$P_i^\mu P_j^\nu R_{\mu\nu} = \frac{1}{2} \left( h_{i,j\rho}^\rho + h_{j,i\rho}^\rho - \partial_\rho \partial^\rho h_{ij} - \partial_i \partial_j (h_\rho^\rho) \right) ,$$

$$\frac{1}{2U_0} P_i^\mu P_j^\nu \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi = -\frac{1}{2} (h_{ti,jt} + h_{tj,it} - h_{ij,tt}) ,$$

$$\partial^\mu \phi \partial^\nu \phi \nabla_\mu \nabla_\nu + 2U_0 \nabla^\rho \nabla_\rho = 2U_0 \partial_k \partial^k ,$$

$$\begin{aligned} P_i^\mu P_j^\nu \left( R_{\mu\nu} - \frac{1}{2U_0} \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi \right) \\ = \frac{1}{2} \left( h_{ki,j}^k + h_{kj,i}^k - h_{ij,k}^k - \partial_i \partial_j (h_\mu^\mu) \right) . \end{aligned}$$

$$\begin{aligned}
S_{2n+2} \rightarrow & \int d^4x \left[ -\frac{1}{8\kappa^2} \left\{ -2h_{tt} \left( \delta^{ij} \partial_k \partial^k - \partial^i \partial^j \right) h_{ij} \right. \right. \\
& + 2h_{ti} \left( \delta^{ij} \partial_k \partial^k - \partial^i \partial^j \right) h_{tj} + h_{ti} \left( 2\delta^{jk} \partial^i - \delta^{ik} \partial^j - \delta^{ij} \partial^k \right) \partial_t h_{jk} \\
& + h_{ij} \left( \left( \delta^{ij} \delta^{kl} - \frac{1}{2} \delta^{ik} \delta^{jl} - \frac{1}{2} \delta^{il} \delta^{jk} \right) \left( -\partial_t^2 + \partial_k \partial^k \right) \right. \\
& \left. \left. - \delta^{ij} \partial^k \partial^l - \delta^{kl} \partial^i \partial^j + \frac{1}{2} \left( \delta^{ik} \partial^j \partial^l + \delta^{il} \partial^j \partial^k + \delta^{jk} \partial^i \partial^l + \delta^{jl} \partial^i \partial^k \right) \right) h_{kl} \right\} \\
& - 2^{2n-2} \alpha U_0^{2n} \left\{ \left( \partial_k \partial^k \right)^n \left( h_{ki,j}^k + h_{kj,i}^k - h_{ij,k}^k - \partial_i \partial_j \left( h_\mu^\mu \right) \right) \right\} \\
& \times \left\{ \left( \partial_k \partial^k \right)^n \left( h_{,k}^{ijk} + h_{,k}^{jik} - h_{,k}^{ij} - \partial^i \partial^j \left( h_\mu^\mu \right) \right) \right\} + U_0 \lambda h_{tt} \Big] ,
\end{aligned}$$

$$\begin{aligned}
S_{2n+3} \rightarrow & \int d^4x \left[ -\frac{1}{8\kappa^2} \left\{ -2h_{tt} \left( \delta^{ij} \partial_k \partial^k - \partial^i \partial^j \right) h_{ij} \right. \right. \\
& + 2h_{ti} \left( \delta^{ij} \partial_k \partial^k - \partial^i \partial^j \right) h_{tj} + h_{ti} \left( 2\delta^{jk} \partial^i - \delta^{ik} \partial^j - \delta^{ij} \partial^k \right) \partial_t h_{jk} \\
& + h_{ij} \left( \left( \delta^{ij} \delta^{kl} - \frac{1}{2} \delta^{ik} \delta^{jl} - \frac{1}{2} \delta^{il} \delta^{jk} \right) \left( -\partial_t^2 + \partial_k \partial^k \right) \right. \\
& \left. \left. - \delta^{ij} \partial^k \partial^l - \delta^{kl} \partial^i \partial^j + \frac{1}{2} \left( \delta^{ik} \partial^j \partial^l + \delta^{il} \partial^j \partial^k + \delta^{jk} \partial^i \partial^l + \delta^{jl} \partial^i \partial^k \right) \right) h_{kl} \right\} \\
& - 2^{2n-1} \alpha U_0^{2n+1} \left\{ \left( \partial_k \partial^k \right)^n \left( h_{ki,j}^k + h_{kj,i}^k - h_{ij,k}^k - \partial_i \partial_j (h_\mu^\mu) \right) \right\} \\
& \times \left\{ \left( \partial_k \partial^k \right)^{n+1} \left( h_{,k}^{ijk} + h_{,k}^{jik} - h_{,k}^{ij}{}^k - \partial^i \partial^j (h_\mu^\mu) \right) \right\} + U_0 \lambda h_{tt} \Big].
\end{aligned}$$

We show that the **only propagating mode is higher derivative graviton** while scalar and vector modes do not propagate.

$$\delta\lambda \Rightarrow h_{tt} = 0.$$

$$\delta h_{tt} \Rightarrow$$

$$\begin{aligned} \lambda &= -\frac{1}{4\kappa^2 U_0} \left( \delta^{ij} \partial_k \partial^k - \partial^i \partial^j \right) h_{ij} \\ &\quad + 2^{2n-1} \alpha U_0^{2n-1} \left( \partial_k \partial^k \right)^{2n} \partial^i \partial^j \left( h_{ki,j}^k + h_{kj,i}^k - h_{ij,k}^k - \partial_i \partial_j (h_\mu^\mu) \right), \end{aligned}$$

or

$$\begin{aligned} \lambda &= -\frac{1}{4\kappa^2 U_0} \left( \delta^{ij} \partial_k \partial^k - \partial^i \partial^j \right) h_{ij} \\ &\quad + 2^{2n} \alpha U_0^{2n} \left( \partial_k \partial^k \right)^{2n+1} \partial^i \partial^j \left( h_{ki,j}^k + h_{kj,i}^k - h_{ij,k}^k - \partial_i \partial_j (h_\mu^\mu) \right). \end{aligned}$$

$$\delta\phi \Rightarrow$$

$$0 = \partial_t \left\{ \lambda + 2^{2n-1} \alpha U_0^{2n-1} \left( \partial_k \partial^k \right)^{2n} \partial^i \partial^j \left( h_{ki,j}^k + h_{kj,i}^k - h_{ij,k}^k - \partial_i \partial_j (h_\mu^\mu) \right) \right\},$$

$$0 = \partial_t \left\{ \lambda + 2^{2n} \alpha U_0^{2n} \left( \partial_k \partial^k \right)^{2n+1} \partial^i \partial^j \left( h_{ki,j}^k + h_{kj,i}^k - h_{ij,k}^k - \partial_i \partial_j (h_\mu^\mu) \right) \right\}.$$

Decomposition of  $h_{ti}$  (shift function  $N_i$ )

$$h_{ti} = \partial_i s + v_i, \quad \partial^i v_i = 0, \quad s: \text{spatial scalar}$$

Linearized diffeomorphism invariance transformations with respect to the spatial coordinates:

$$\delta x^i = \partial^i u + w^i, \quad \partial_i w^i = 0 \Rightarrow \delta s = \partial_t u, \quad \delta v_i = \partial_t w_i,$$

Gauge fixing condition  $s = v^i = 0 \Rightarrow h_{ti} = 0$

$$\delta h_{ti} \Rightarrow \partial_t \left( -2\delta^{jk} \partial^i + \delta^{ik} \partial^j + \delta^{ij} \partial^k \right) h_{jk} = 0,$$

identical with that in the Einstein gravity  
(not including higher derivative terms)

## Decomposition of $h_{ij}$

$$h_{ij} = \delta_{ij}A + \partial_j B_i + \partial_i B_j + C_{ij} + \left( \partial_i \partial_j - \frac{1}{3} \delta_{ij} \partial_k \partial^k \right) E ,$$

$$\partial^i B_i = 0 , \quad \partial^i C_{ij} = \partial^j C_{ij} = 0 , \quad C_i^i = 0 ,$$

$$\delta h_{ti} \Rightarrow 0 = \partial_t \left( -4\partial_i A + 2\partial_k \partial^k B_i + \frac{4}{3} \partial_i \partial_k \partial^k E \right) \dots (*)$$

$$\partial^i \times \Rightarrow \partial_t \partial_i \partial^i \left( -4A + \frac{4}{3} \partial_k \partial^k E \right) = 0 \Rightarrow A = \frac{1}{3} \partial_k \partial^k E \dots (**)$$

by assuming  $A, E \rightarrow 0$  at spatial infinity.

$$(*), (**) \Rightarrow \partial_t \partial_j \partial^j B_i = 0 \Rightarrow B_i = 0$$

by assuming  $B_i \rightarrow 0$  at spatial infinity.

$\delta h_{tt}$  and  $\delta\phi$  equations:

$$\lambda = \frac{1}{2\kappa^2 U_0} \partial_k \partial^k \left( -A + \frac{1}{3} \partial_j \partial^j E \right) - 2^{2n} \alpha U_0^{2n-1} \left( \partial_k \partial^k \right)^{2n+2} \left( -A + \frac{1}{3} \partial_j \partial^j E \right),$$
$$0 = \partial_t \left\{ \lambda + 2^{2n} \alpha U_0^{2n-1} \left( \partial_k \partial^k \right)^{2n+2} \left( -A + \frac{1}{3} \partial_j \partial^j E \right) \right\},$$

or

$$\lambda = \frac{1}{2\kappa^2 U_0} \partial_k \partial^k \left( -A + \frac{1}{3} \partial_j \partial^j E \right) - 2^{2n+1} \alpha U_0^{2n} \left( \partial_k \partial^k \right)^{2n+3} \left( -A + \frac{1}{3} \partial_j \partial^j E \right),$$
$$0 = \partial_t \left\{ \lambda + 2^{2n+1} \alpha U_0^{2n} \left( \partial_k \partial^k \right)^{2n+3} \left( -A + \frac{1}{3} \partial_j \partial^j E \right) \right\}.$$

$$\Rightarrow \lambda = 0 \text{ since } A = \frac{1}{3} \partial_k \partial^k E$$

Scalar modes  $\lambda$  and the vector mode  $B_i$  do not propagate.

$\delta A \Rightarrow$

$$0 = +\frac{1}{8\kappa^2} \left\{ -12 \left( -\partial_t^2 + \partial_k \partial^k \right) A + 8 \partial_k \partial^k A + \frac{4}{3} \left( \partial_k \partial^k \right)^2 E \right\} \\ - 2^{2n-1} \alpha U_0^{2n} \left( -\partial_i \partial_j - \delta_{ij} \partial_k \partial^k \right) \left\{ \left( \partial_k \partial^k \right)^{2n} \right. \\ \left. \times \left( -\partial^i \partial^j A - \delta^{ij} \partial_k \partial^k A + \frac{1}{3} \partial^i \partial^j \partial_k \partial^k E + \frac{1}{3} \delta^{ij} \left( \partial_k \partial^k \right)^2 E \right) \right\} ,$$

or

$$0 = +\frac{1}{8\kappa^2} \left\{ -12 \left( -\partial_t^2 + \partial_k \partial^k \right) A + 8 \partial_k \partial^k A + \frac{4}{3} \left( \partial_k \partial^k \right)^2 E \right\} \\ - 2^{2n} \alpha U_0^{2n+1} \left( -\partial_i \partial_j - \delta_{ij} \partial_k \partial^k \right) \left\{ \left( \partial_k \partial^k \right)^{2n+1} \right. \\ \left. \times \left( -\partial^i \partial^j A - \delta^{ij} \partial_k \partial^k A + \frac{1}{3} \partial^i \partial^j \partial_k \partial^k E + \frac{1}{3} \delta^{ij} \left( \partial_k \partial^k \right)^2 E \right) \right\} ,$$



$\delta E \Rightarrow$

$$0 = \partial_k \partial^k \left[ \frac{1}{8\kappa^2} \left\{ \frac{4}{3} \left( -\partial_t^2 + \partial_k \partial^k \right) \partial_k \partial^k E + \frac{4}{3} \partial_k \partial^k A + \frac{16}{9} \left( \partial_k \partial^k \right)^2 E \right\} \right. \\ \left. + \frac{2^{2n-1}}{3} \alpha U_0^{2n} \left( -\partial_i \partial_j - \delta_{ij} \partial_k \partial^k \right) \left\{ \left( \partial_k \partial^k \right)^{2n} \right. \right. \\ \left. \left. \times \left( -\partial^i \partial^j A - \delta^{ij} \partial_k \partial^k A + \frac{1}{3} \partial^i \partial^j \partial_k \partial^k E + \frac{1}{3} \delta^{ij} \left( \partial_k \partial^k \right)^2 E \right) \right\} \right],$$

or

$$0 = \partial_k \partial^k \left[ \frac{1}{8\kappa^2} \left\{ \frac{4}{3} \left( -\partial_t^2 + \partial_k \partial^k \right) \partial_k \partial^k E + \frac{4}{3} \partial_k \partial^k A + \frac{16}{9} \left( \partial_k \partial^k \right)^2 E \right\} \right. \\ \left. + \frac{2^{2n}}{3} \alpha U_0^{2n+1} \left( -\partial_i \partial_j - \delta_{ij} \partial_k \partial^k \right) \left\{ \left( \partial_k \partial^k \right)^{2n+1} \right. \right. \\ \left. \left. \times \left( -\partial^i \partial^j A - \delta^{ij} \partial_k \partial^k A + \frac{1}{3} \partial^i \partial^j \partial_k \partial^k E + \frac{1}{3} \delta^{ij} \left( \partial_k \partial^k \right)^2 E \right) \right\} \right],$$

$$\Rightarrow 0 = \partial_t^2 A \left( \text{using } A = \frac{1}{3} \partial_k \partial^k E \right) \Rightarrow A = E = 0.$$

All the scalar modes  $\phi$ ,  $\lambda$ ,  $h_{tt}$ ,  $s$ ,  $A$ , and  $E$

and all the vector modes  $v_i$  and  $B_i$  do not propagate.

The only propagating mode is massless graviton  $C_{ij}$ .

( $\Leftrightarrow$  Hořava quantum gravity)

$$\begin{aligned}
 S_{2n+2} &= \int d^4x \left[ \frac{1}{8\kappa^2} \left\{ C_{ij} \left( -\partial_t^2 + \partial_k \partial^k \right) C^{ij} \right\} \right. \\
 &\quad \left. - 2^{2n-2} \alpha U_0^{2n} \left\{ \left( \partial_k \partial^k \right)^{n+1} C_{ij} \right\} \left\{ \left( \partial_k \partial^k \right)^{n+1} C^{ij} \right\} \right], \\
 S_{2n+3} &= \int d^4x \left[ \frac{1}{8\kappa^2} \left\{ C_{ij} \left( -\partial_t^2 + \partial_k \partial^k \right) C^{ij} \right\} \right. \\
 &\quad \left. - 2^{2n-1} \alpha U_0^{2n+1} \left\{ \left( \partial_k \partial^k \right)^{n+1} C_{ij} \right\} \left\{ \left( \partial_k \partial^k \right)^{n+2} C^{ij} \right\} \right].
 \end{aligned}$$

Propagator:

$$\begin{aligned}
 \langle h_{ij}(p) h_{kl}(-p) \rangle &= \langle C_{ij}(p) C_{kl}(-p) \rangle \\
 &= \frac{1}{2} \left\{ \left( \delta_{ij} - \frac{p_i p_j}{p^2} \right) \left( \delta_{kl} - \frac{p_k p_l}{p^2} \right) - \left( \delta_{ik} - \frac{p_i p_k}{p^2} \right) \left( \delta_{jl} - \frac{p_j p_l}{p^2} \right) \right. \\
 &\quad \left. - \left( \delta_{il} - \frac{p_i p_l}{p^2} \right) \left( \delta_{jk} - \frac{p_j p_k}{p^2} \right) \right\} \\
 &\times \begin{cases} (p^2 - 2^{2n} \alpha \kappa^2 U_0^{2n} p^{4(n+1)})^{-1}, & z = 2n + 2 \text{ case} \\ (p^2 - 2^{2n-1} \alpha \kappa^2 U_0^{2n+1} p^{2(2n+3)})^{-1}, & z = 2n + 3 \text{ case} \end{cases} .
 \end{aligned}$$

$$p^2 = \sum_{i=1}^3 (p^i)^2, \quad p^2 = - (p^0)^2 + \mathbf{p}^2.$$

Assume  $\alpha < 0$  in order to avoid tachyon pole.

In the ultraviolet region, ( $\mathbf{k}$ ), Propagator behaves

$\sim 1/|\mathbf{p}|^4$  for  $z = 2$  ( $n = 0$ ) case

$\sim 1/|\mathbf{p}|^6$  for  $z = 3$  ( $n = 0$ ) case: power-counting renormalizable.

$z = 2n+2$  ( $n \geq 1$ ) or  $z = 2n+3$  ( $n \geq 1$ ) case: super-renormalizable.

Dispersion relation  $\omega \propto k^z$ ,

$\omega$  : angular frequency corresponding to the energy

$$\Rightarrow \text{phase speed } v_p = \frac{\omega}{k} = \alpha c_0 k^{z-1}$$

$$\text{group speed } v_g = \frac{d\omega}{dk} = \alpha c_0 z k^{z-1}$$

For large  $k$ ,  $v_p, v_g > \text{light speed}$ .

Note 1: Four momentum  $\sim (p^0, \mathbf{p})$  is time-like.

$$p^0 = \sqrt{\mathbf{p}^2 - 2^{2n} \alpha \kappa^2 U_0^{2n} \mathbf{p}^{4(n+1)}} \text{ or } \sqrt{\mathbf{p}^2 - 2^{2n-1} \alpha \kappa^2 U_0^{2n+1} \mathbf{p}^{2(2n+3)}}$$

Note 2: No violation of causality.

Diagonal symmetry is not broken:

$$\phi_0 = -\sqrt{2U_0}t_0$$

similar structure  $\sim$  ghost condensation model

N. Arkani-Hamed, H.C. Cheng, M.A. Luty and S. Mukohyama,

“Ghost condensation and a consistent infrared modification of gravity”

arXiv:hep-th/0312099

$$S_{\text{ghost}} = \int d^4x \sqrt{-g} P(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi)$$

# FRW cosmology

Start with a little bit general action:

$$\begin{aligned}
 S = & \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} - \sum_{n=0}^{n_{\max}} \alpha_n \left\{ (\partial^\mu \phi \partial^\nu \phi \nabla_\mu \nabla_\nu - \partial_\mu \phi \partial^\mu \phi \nabla^\rho \nabla_\rho)^n P_\alpha^\mu P_\beta^\nu \right. \right. \\
 & \times \left. \left( R_{\mu\nu} - \frac{1}{2U_0} \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi \right) \right\} \\
 & \times \left\{ (\partial^\mu \phi \partial^\nu \phi \nabla_\mu \nabla_\nu - \partial_\mu \phi \partial^\mu \phi \nabla^\rho \nabla_\rho)^n P^{\alpha\mu} P^{\beta\nu} \left( R_{\mu\nu} - \frac{1}{2U_0} \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi \right) \right\} \\
 & - \sum_{m=0}^{m_{\max}} \tilde{\alpha} \left\{ (\partial^\mu \phi \partial^\nu \phi \nabla_\mu \nabla_\nu - \partial_\mu \phi \partial^\mu \phi \nabla^\rho \nabla_\rho)^m P_\alpha^\mu P_\beta^\nu \right. \\
 & \times \left. \left( R_{\mu\nu} - \frac{1}{2U_0} \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi \right) \right\} \\
 & \times \left\{ (\partial^\mu \phi \partial^\nu \phi \nabla_\mu \nabla_\nu - \partial_\mu \phi \partial^\mu \phi \nabla^\rho \nabla_\rho)^{m+1} P^{\alpha\mu} P^{\beta\nu} \left( R_{\mu\nu} - \frac{1}{2U_0} \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi \right) \right\} \\
 & \left. - \lambda \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + U_0 \right) \right].
 \end{aligned}$$

In low energy, Einstein-Hilbert term +  $n = 0$  term could dominate:

$$\begin{aligned}
 S \sim & \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} - \alpha_0 P_\alpha^\mu P_\beta^\nu \left( R_{\mu\nu} - \frac{1}{2U_0} \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi \right) \right. \\
 & \times P^{\alpha\mu} P^{\beta\nu} \left( R_{\mu\nu} - \frac{1}{2U_0} \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi \right) \\
 & \left. - \lambda \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + U_0 \right) \right].
 \end{aligned}$$

FRW metric:

$$ds^2 = -e^{2b(t)} dt^2 + a(t)^2 \sum_{i=1,2,3} (dx^i)^2.$$



FRW equation:

$$\frac{3}{\kappa^2} H^2 + 81\alpha_0 H^4 = \rho_{\text{matter}} .$$

$\rho_{\text{matter}}$ : energy-density of the matter

1st order differential equation with respect to the scale factor  $a(t)$

If  $\alpha_0 < 0$  and  $\rho_{\text{matter}} = 0$ , de Sitter solution

$$H^2 = -\frac{1}{27\alpha_0\kappa^2} .$$

Inflation in the early universe?

# Superluminal neutrinos

OPERA experiment:

OPERA Collaboration,

“Measurement of the neutrino velocity with the OPERA detector in the CNGS beam”, arXiv:1109.4897 [hep-ex]

⇒

S.N., S. D. Odintsov,

“Could the dynamical Lorentz symmetry breaking induce the superluminal neutrinos?” Eur.Phys.J. C71 (2011) 1801, arXiv:1110.0889 [hep-ph].

Model:

$$S = \int d^4x \left[ \bar{\psi} \left\{ \gamma^\mu \partial_\mu + \alpha (P_\mu^\nu \gamma^\mu \partial_\nu)^{2n+1} \right\} \psi - \lambda \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + U_0 \right) \right].$$

⇒ Dirac equation

$$0 = \left\{ \gamma^\mu \partial_\mu + \alpha (P_\mu^\nu \gamma^\mu \partial_\nu)^{2n+1} \right\} \psi$$

$$\Rightarrow 0 = \left\{ \gamma^0 \partial_0 + \gamma^i \partial_i + \alpha (\gamma^i \partial_i)^{2n+1} \right\} \psi$$

⇒ dispersion relation:  $\omega = k \sqrt{1 + 2\alpha k^{2n} + \alpha^2 k^{4n}}$ .

$\omega$ : angular frequency,  $k$ : wave number

In the high energy region,  $\omega \sim |\alpha| k^{2n+1}$

phase velocity:  $v_p \equiv \frac{\omega}{k} = |\alpha| k^{2n}$ , group velocity:  $v_g \equiv \frac{d\omega}{dk} = (2n + 1) |\alpha| k^{2n}$

When  $k$  becomes larger, both of  $v_p$  and  $v_g$  become also larger in an unbounded way and exceed the light speed.

# Summary

- We formulated covariant higher derivative gravity with Lagrange multiplier constraint and scalar projectors.
  - The theory admits flat space solution.
  - Its gauge-fixing formulation is fully developed.
  - The only propagating mode is (higher derivative) graviton, while scalar and vector modes do not propagate.
  - The theory could be power-counting renormalizable.

- The preliminary study of FRW cosmology indicates to the possibility of inflationary universe solution.
  - 1st FRW equation in the theory turns out to be the first order differential equation which is quite unusual for higher derivative gravity which normally leads to third order differential equation with respect to scale factor.
- Superluminal neutrinos?