多面体を裏返す

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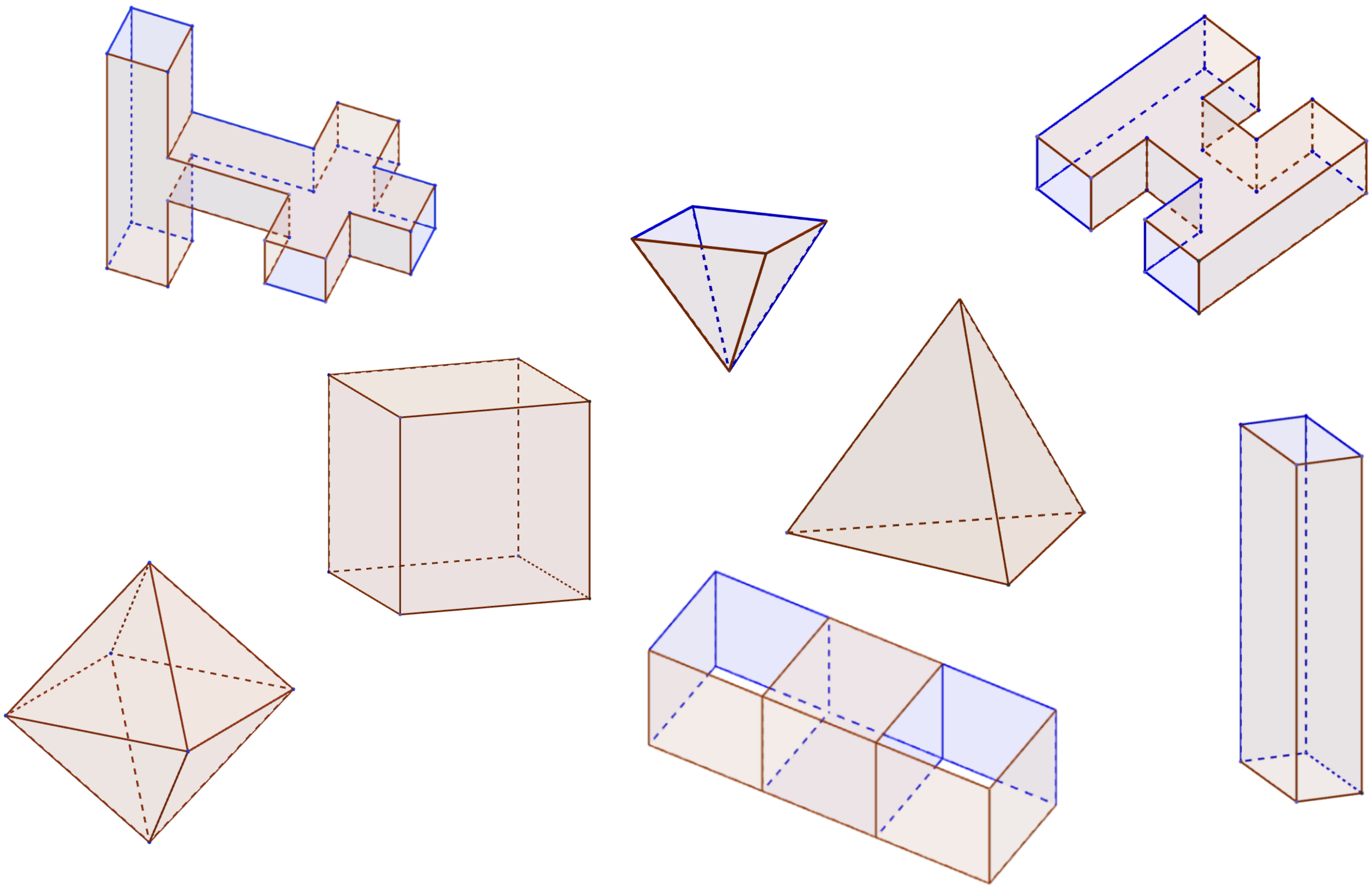
堀尾直史氏との共同研究

Reversible?



Hiroshi Maehara defined an origami deformation of a polyhedral surface M in R^3 as the existence of continuous motion f_t: M → R^3 ($0 \le t \le 1$) of M such that

(1) f_0 is a inclusion map,
(2) for each face of M, the induced motion of the face is a rigid motion,
(3) two faces may touch or overlap during the motion, but they never go through each other,
(4) the motion is not a rigid motion of the whole M.

He also called a polyhedral surface subdivision-reversible (shortly s-reversible). He proved several results and many interesting problems.

Reference [1] H.Maehara: Reversing a polyhedral surface by origami-deformation, European Journal of Combinatorics 31 (2010), 1171-1180

Maehara's theorem

Every rectangular tube is s-reversible.

（[1] Th.3）

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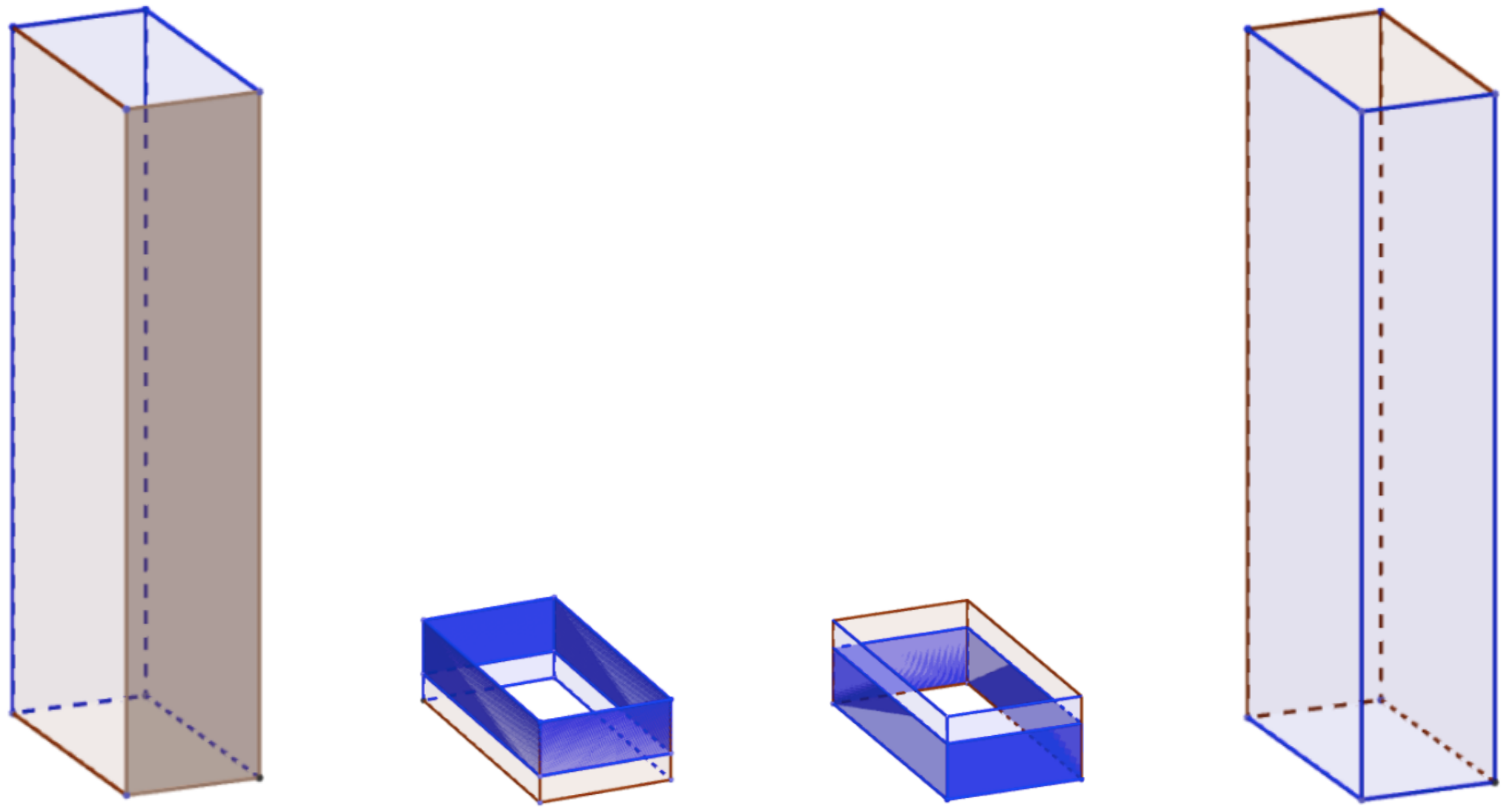
([1] Th.3)

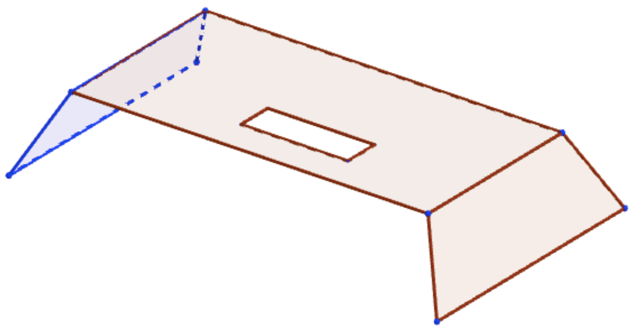


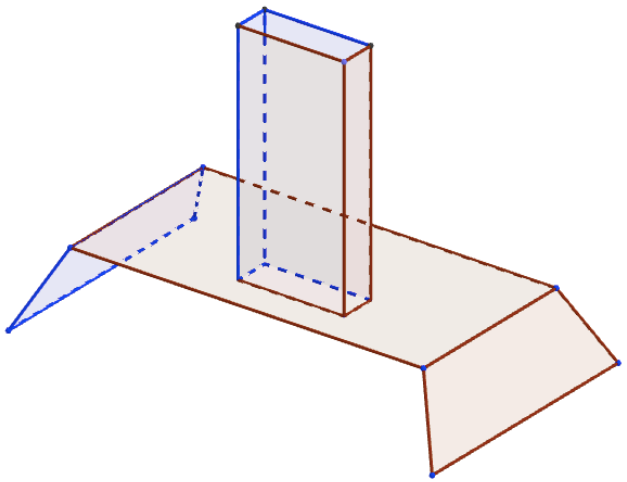
Maehara’s theorem

The surface obtained from a s-reversible polyhedral surface M by applying a tube-attachment operation is also s-reversible.

（[1] Th.4）

a tube-attachment operation





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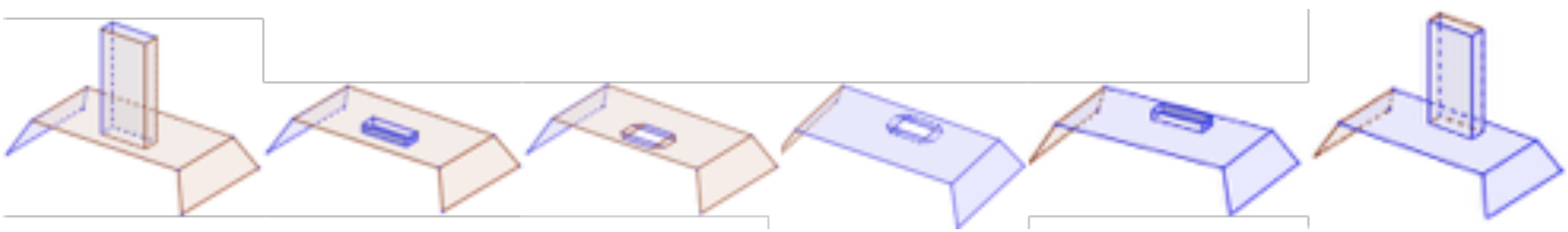
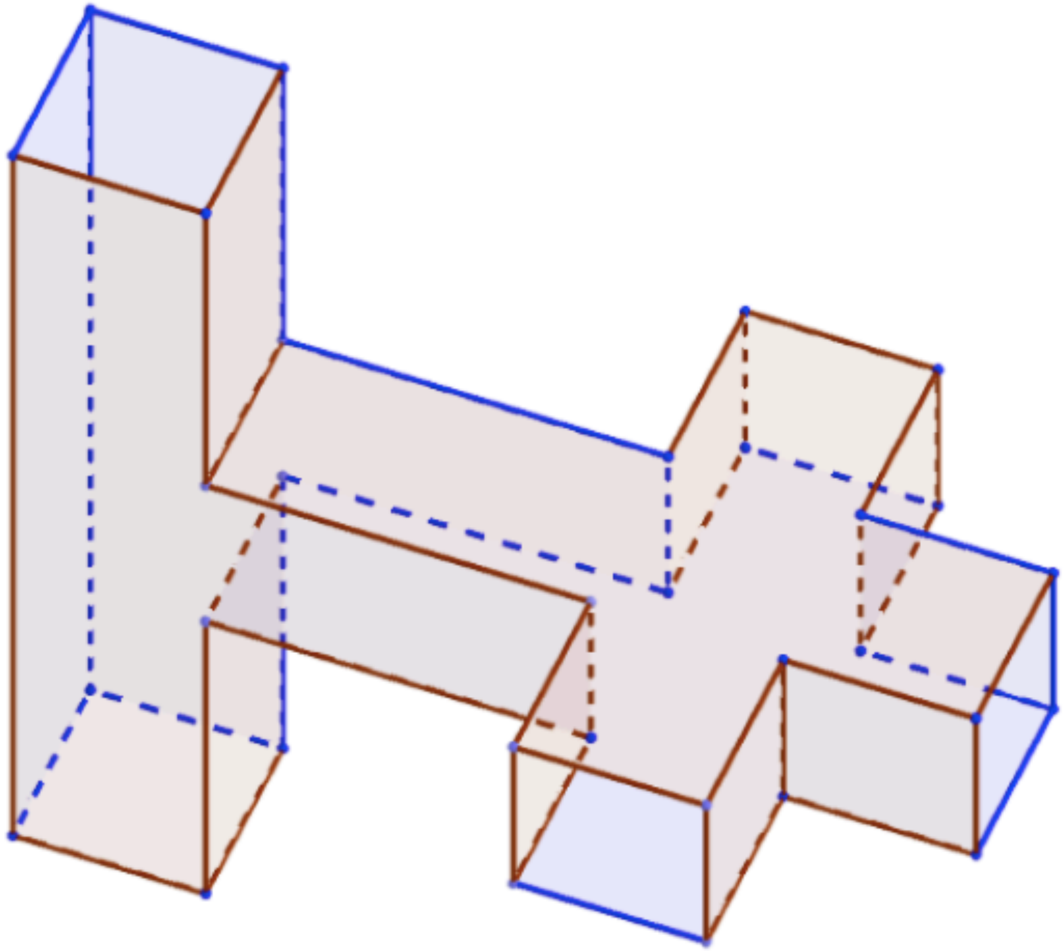
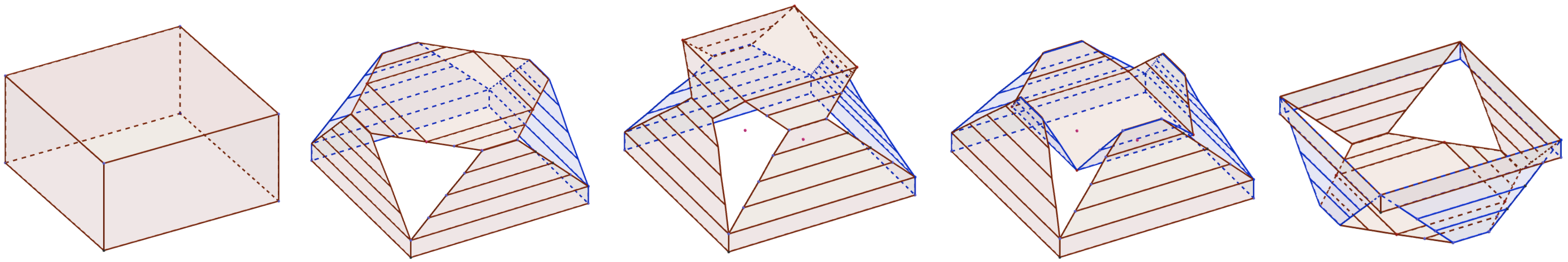


If polyhedral surface have not the vertex of convex point and have subdivided origami-deformation to a tube, then it has a possibility to be s-reversible.

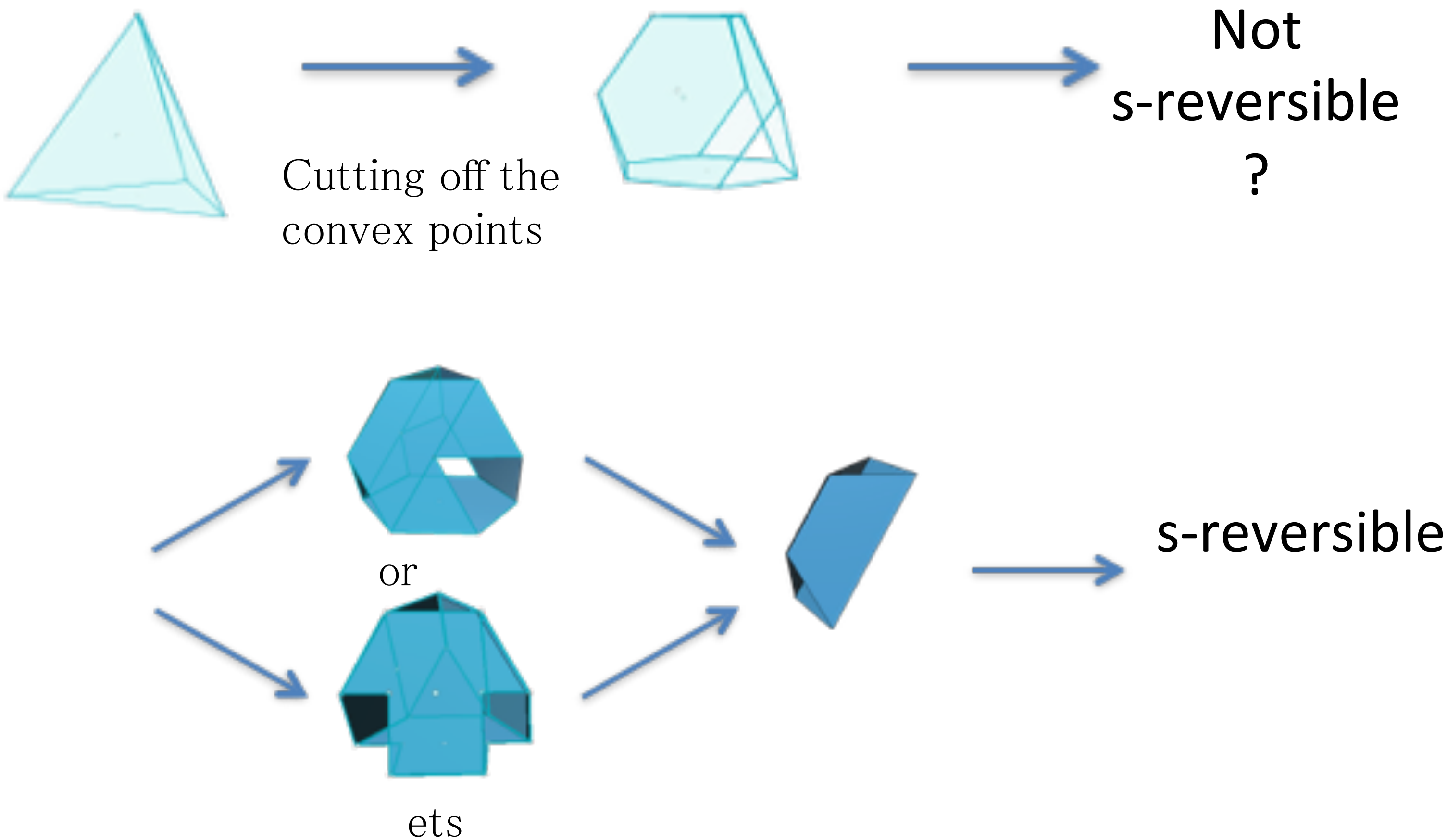
s-reversible !



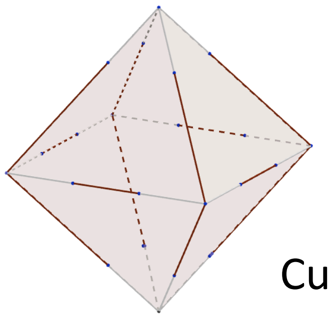
s-reversible !



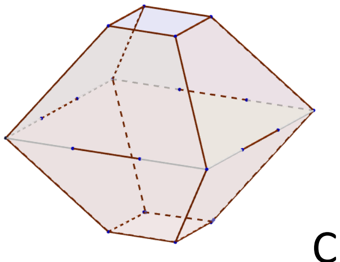
Regular tetrahedron



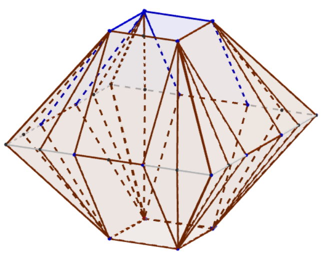
Regular octahedron



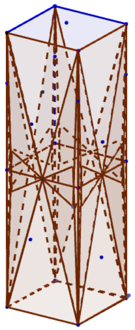
Cutting off one convex vertex and the opposite vertex



Cutting along edges around the other vertices

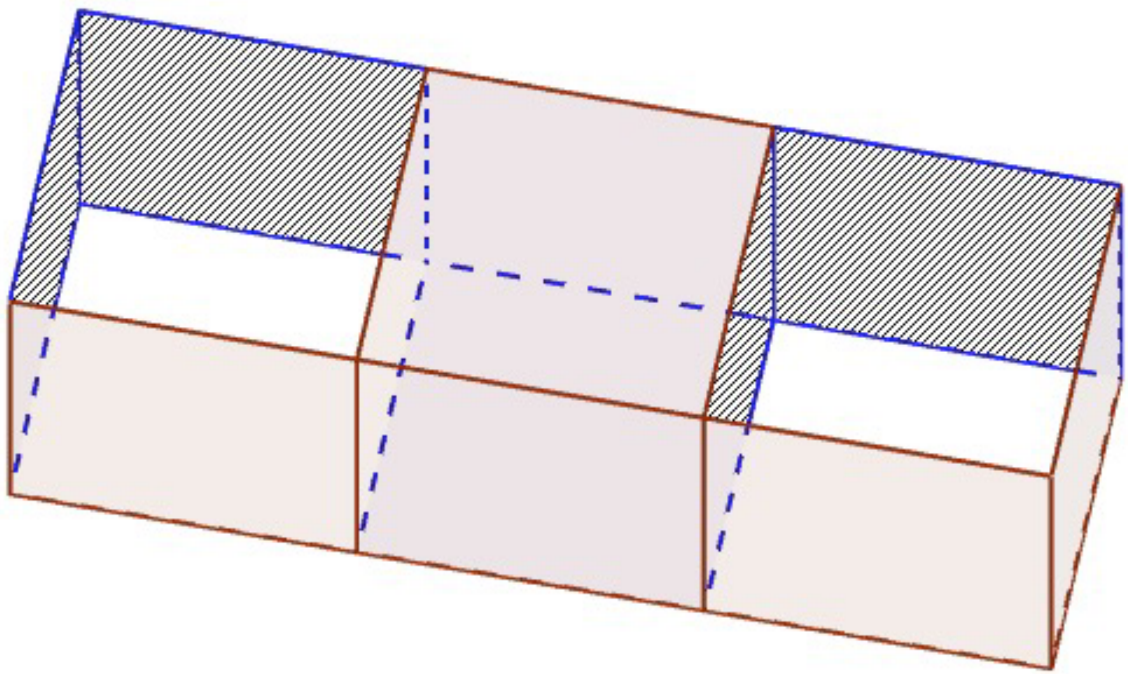


Make pleats and fold in, leaving the part to become the side of the tube.



s-reversible

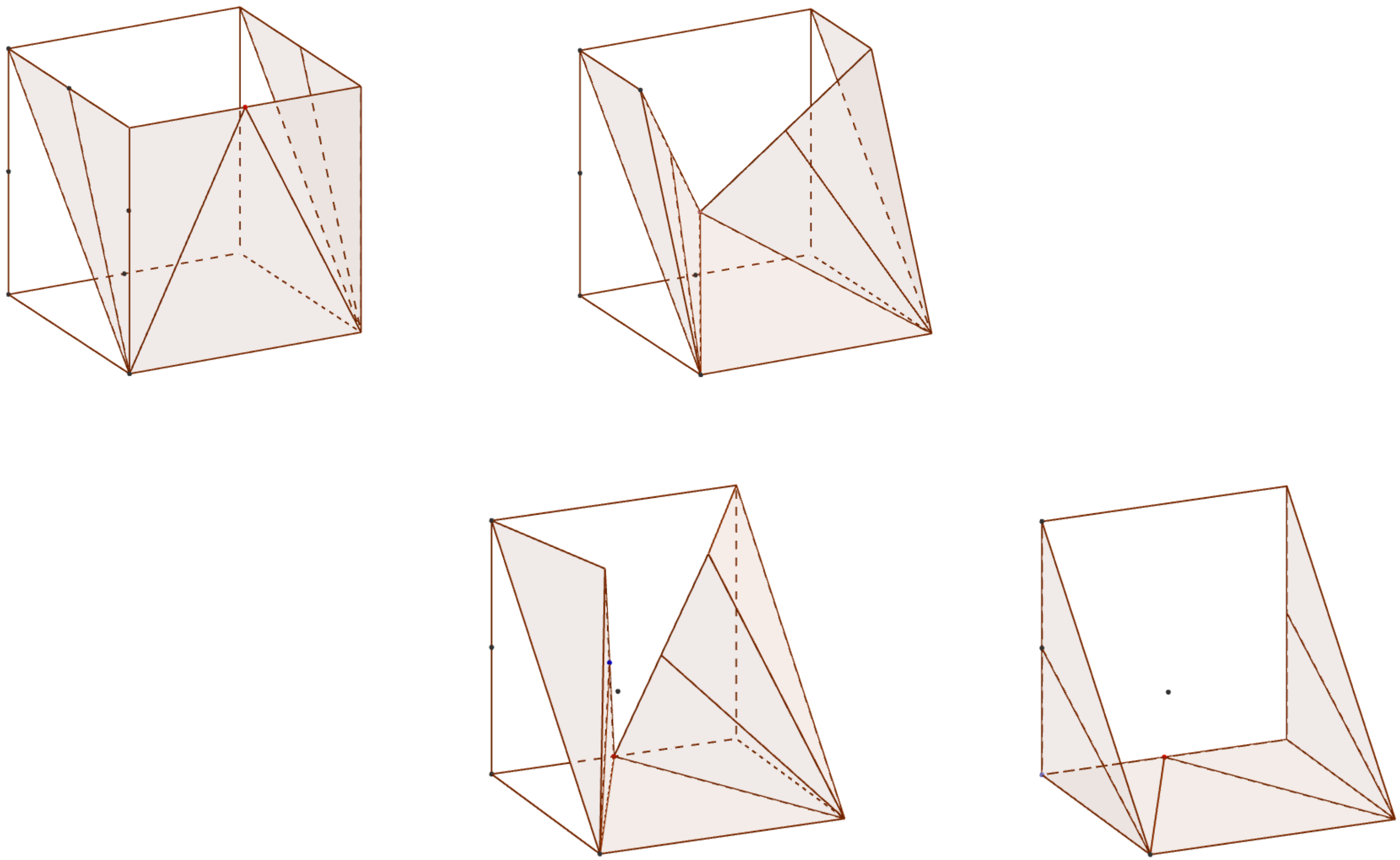
reverse this polyhedral surface



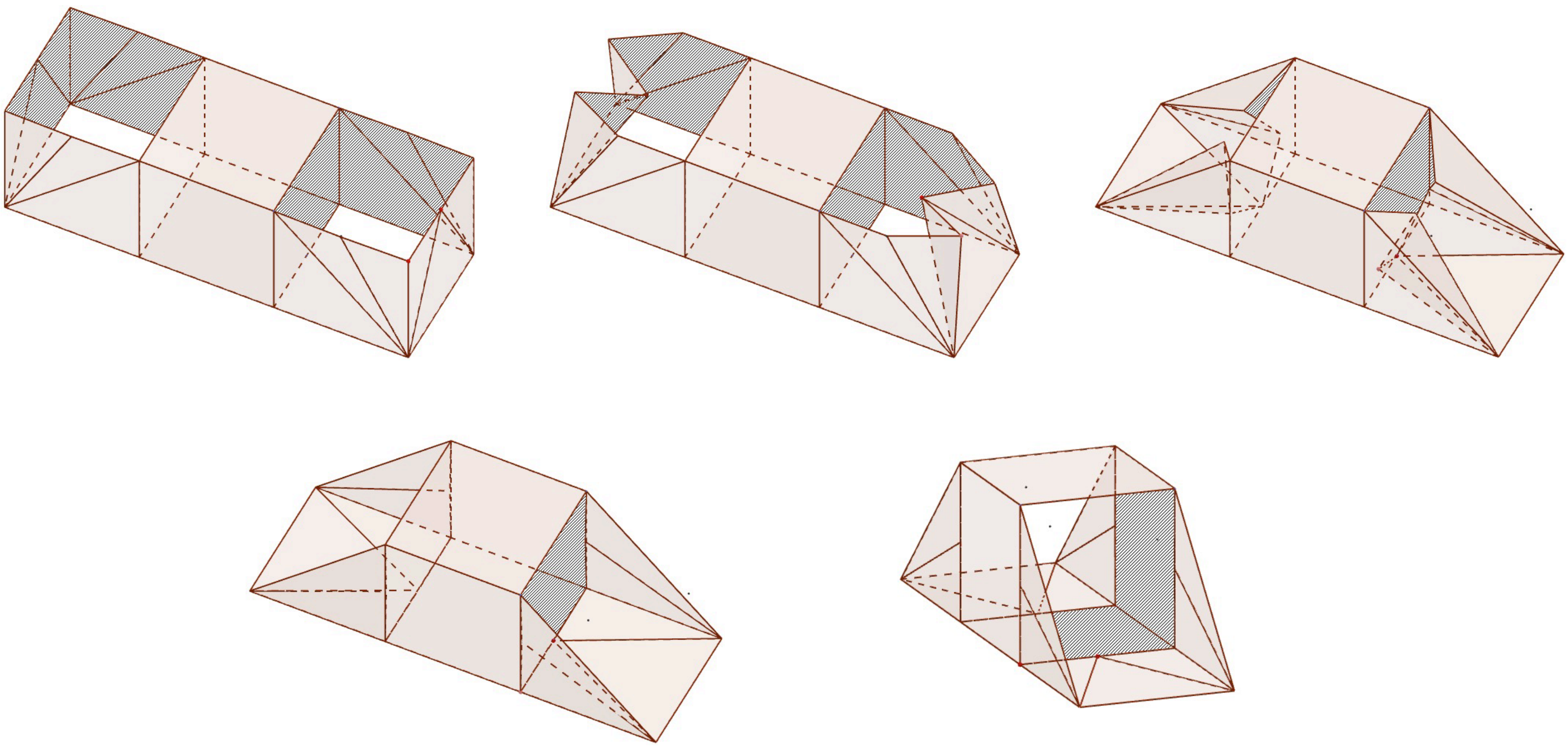
XC_4 : a 3 times extended cube with 4 square holes

This is not a tube. At first we need to deform it to a tube.

New operation：<u>semi-flattening-operation</u>



Applying semi-flattering operation to a XC_4.



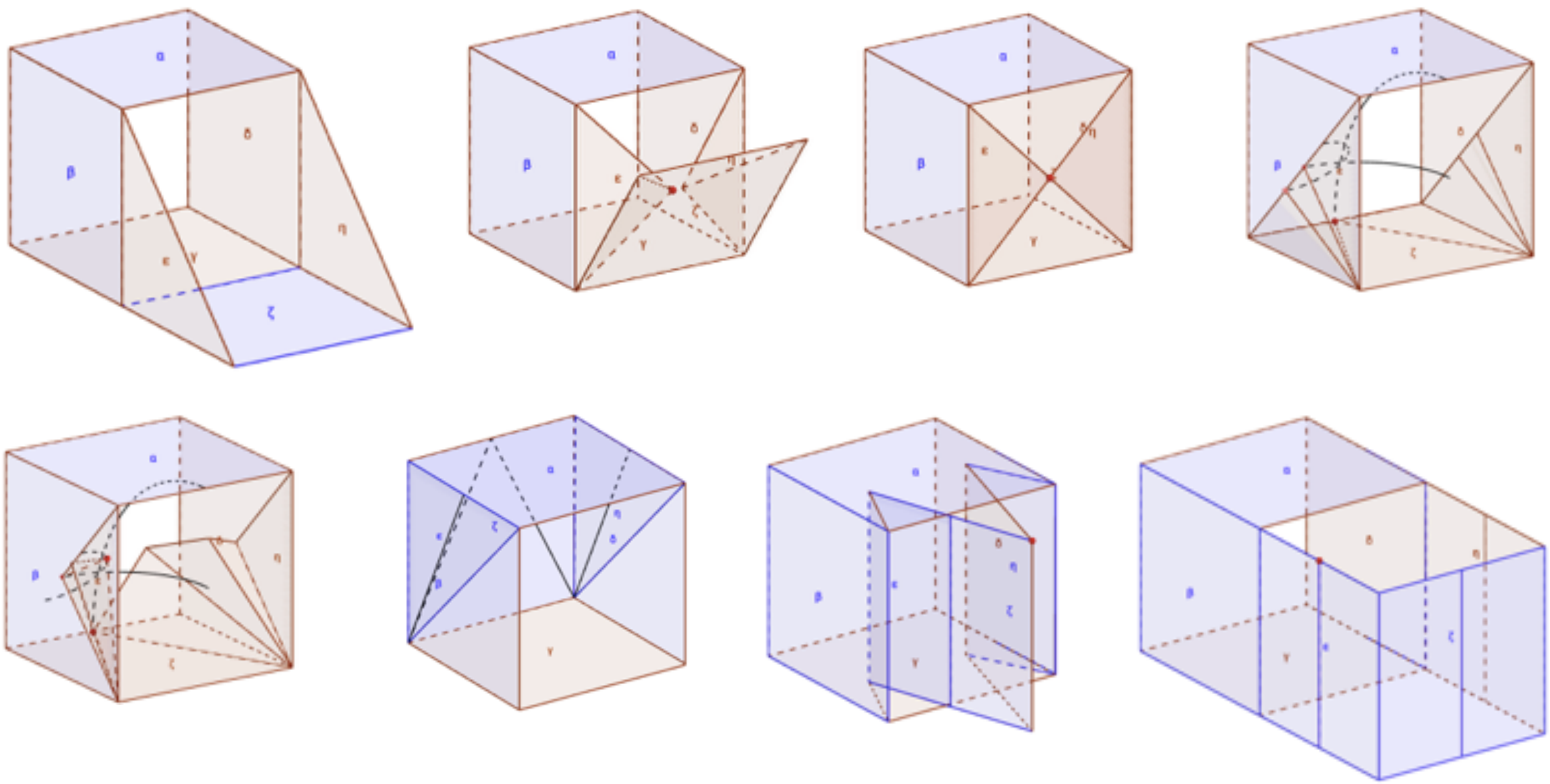
Now a XC_4 deforms to a tube.

Remark

If we can deform a polyhedral surface into a rectangular tube, we can apply Maehara's method, but it does not mean reversing can be completed.

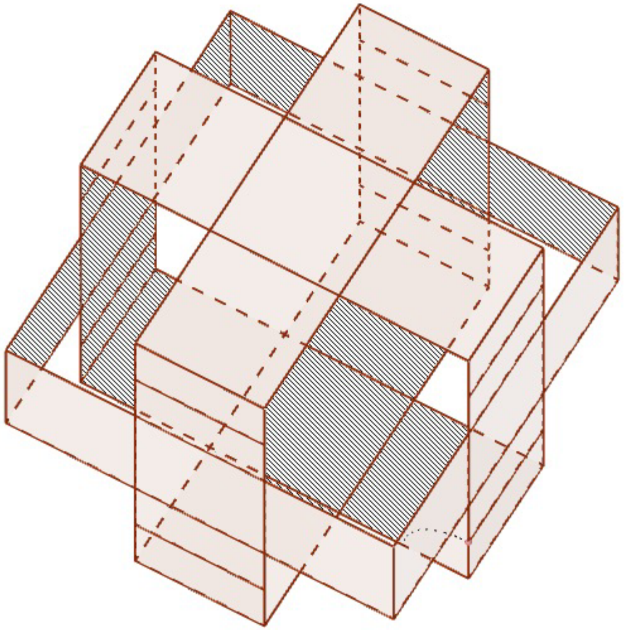
After the Maehara's method

Only one side is shown. Red are front surfaces and blue are back surfaces.



A XC_4 is s-reversible.

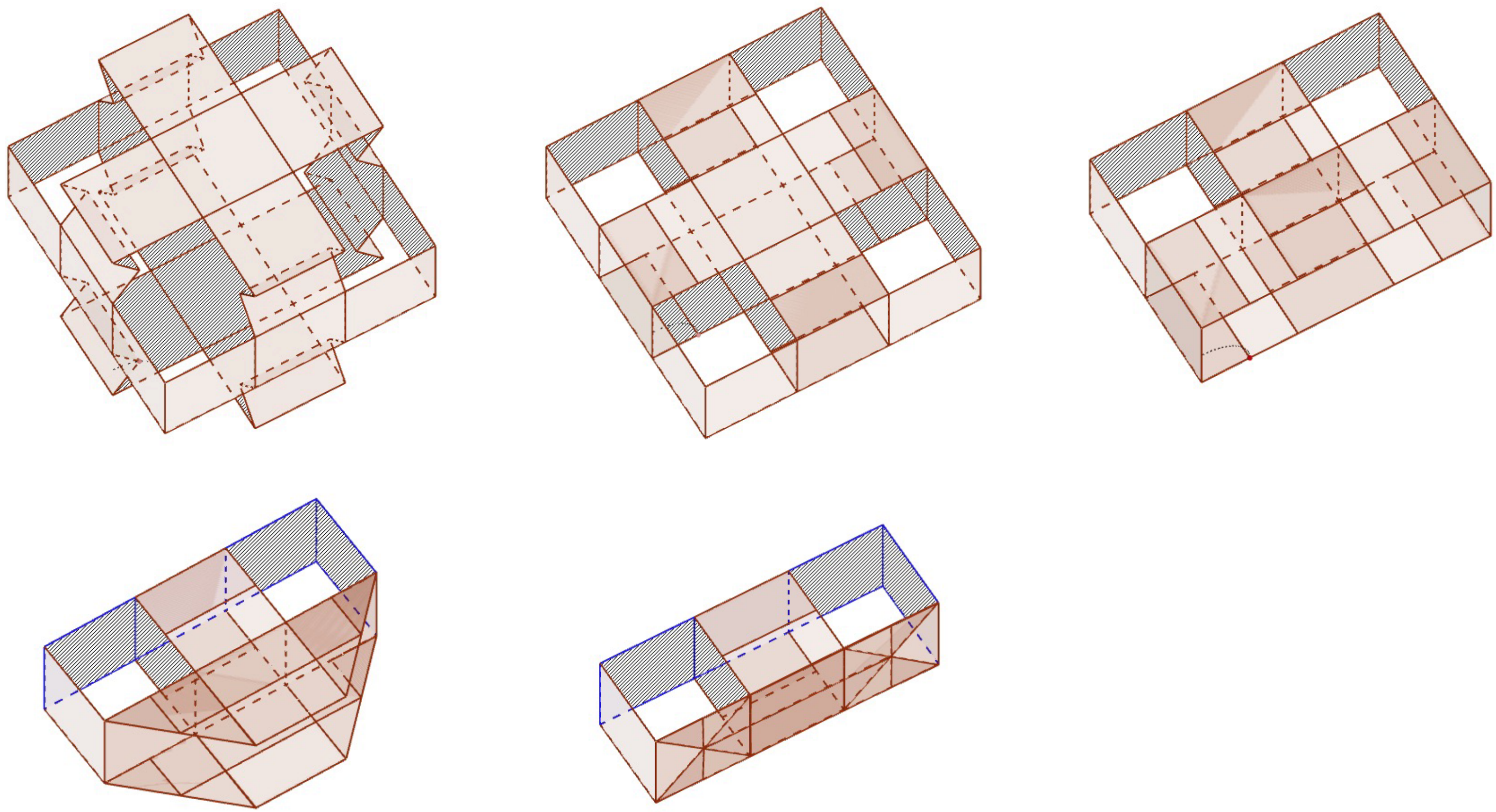
reverse this polyhedral surface



C_8 : a surface of cube with 8 holes

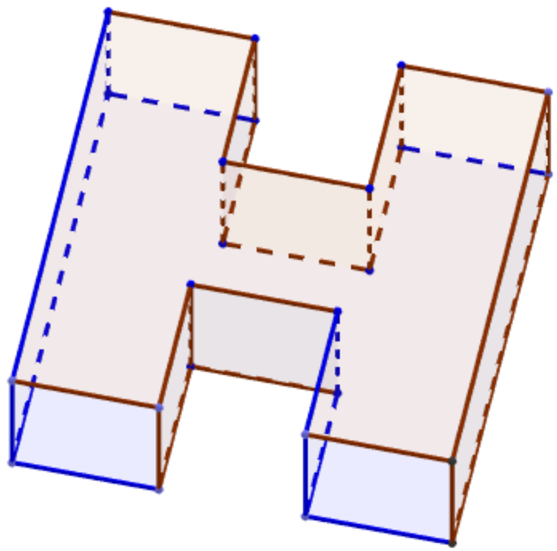
This is not a tube. At first we need to deform it to a tube.

Applying semi-flattering operation to a C_8.

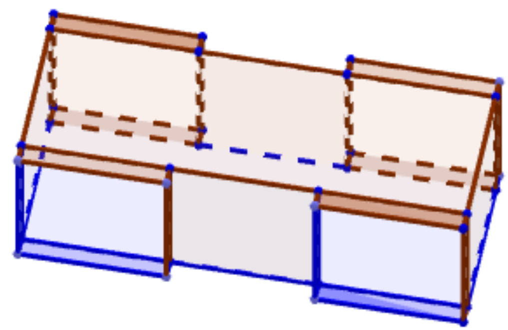


Now a C_8 deforms to a XC_4, then deforms to a tube. Even taking into account the folded faces, this is s-reversible.

reverse this polyhedral surface



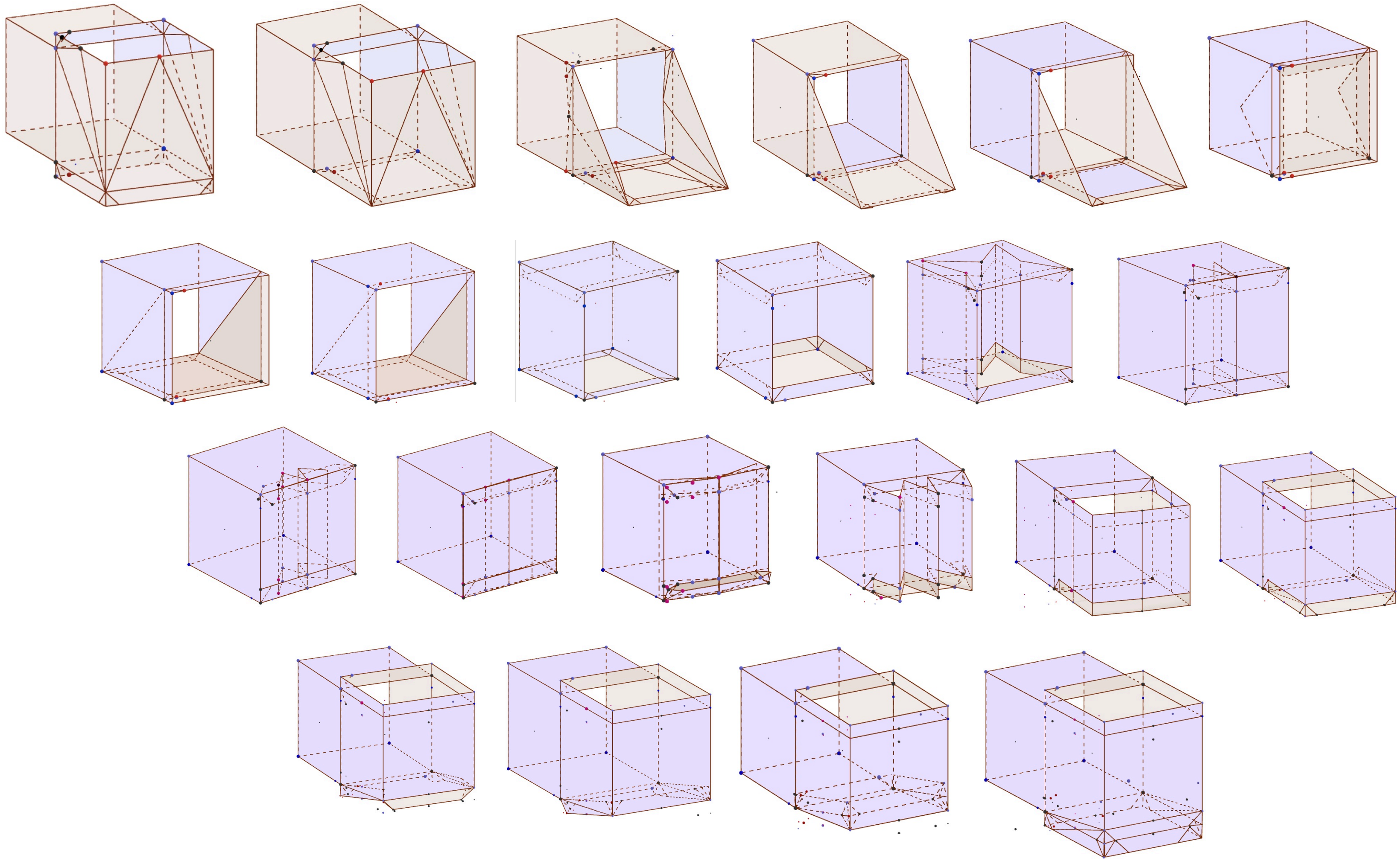
H_4 : a H-shaped polyhedral surface with 4 holes



We cannot deform this polyhedral surface to a XC_4.
No matter how hard we try to shorten the hights of the tubes, we cannot make the hights zero.
We need another solution.

Reversing a H_4.

Only one side is shown. Red are front surfaces and blue are back surfaces.



Now a H_4 is s-reversible.

Considering further

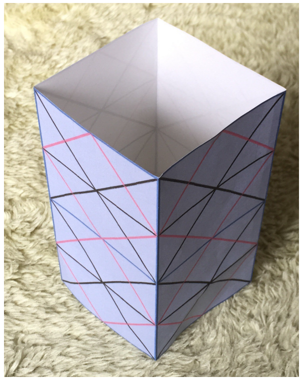
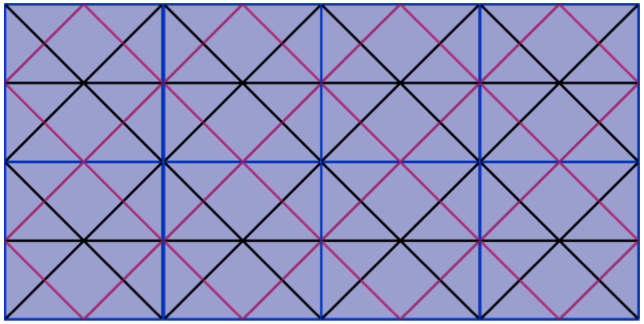
cubical-tube-unit-attachment operation

Consider polyhedral surfaces made of unit cubes on the grid points. Take a initial unit cube. Attach the cubical-tube-unit on some unit cube of a previous polyhedral surface such that the attaching unit-tube does not overlap the other unit cubes of the previous polyhedral surface, and remove the attached face of the previous polyhedral surface.

Theorem

Every polyhedral surface made of cubical-tube-unit-attachment operation is s-reversible.

double flexatube

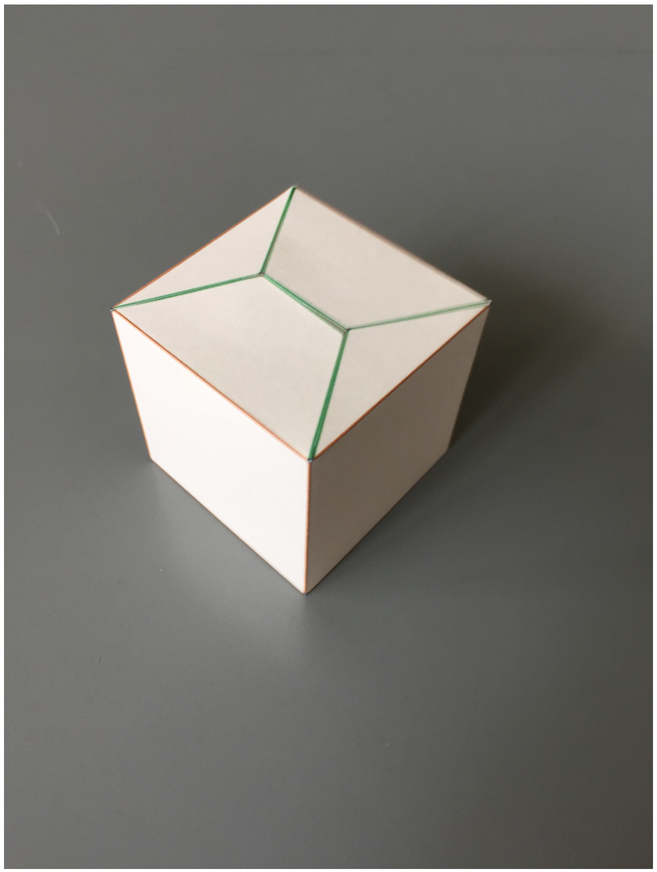
 

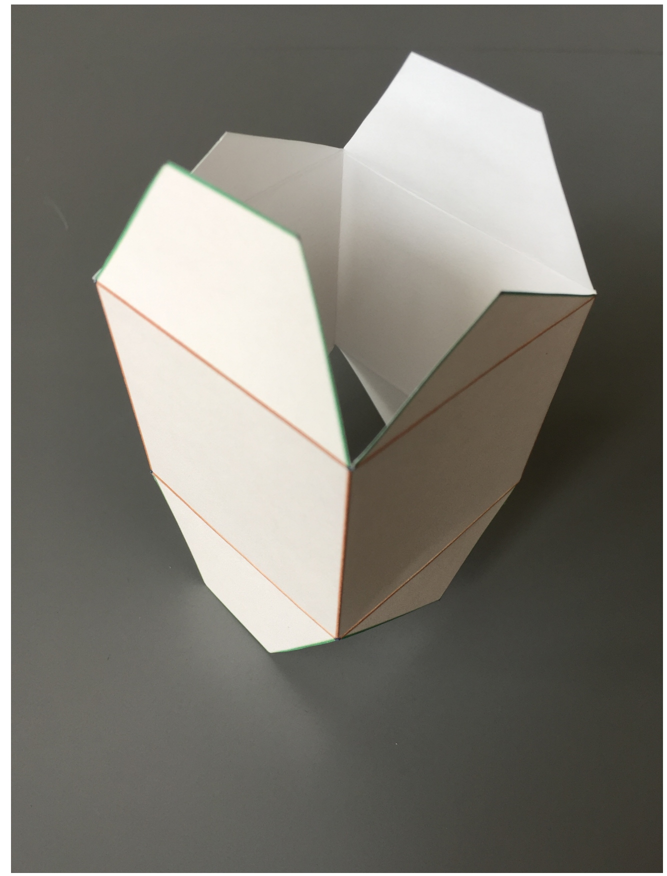
double flexatube and its development

Def.1 From an extended box 1×1×2, remove a pair of opposite faces, and crease remaining four face as shown the above figure, which is consists of 96 right isosceles triangles and 16 squares.

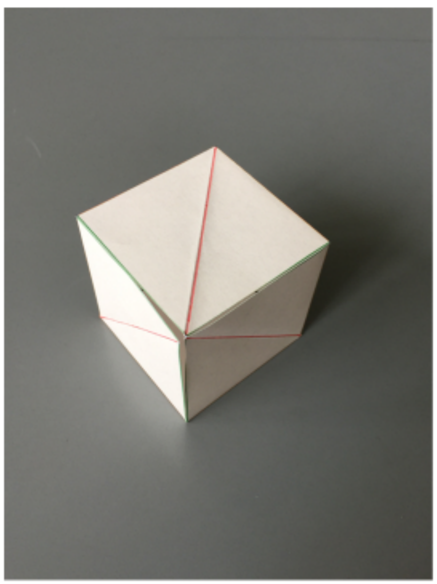
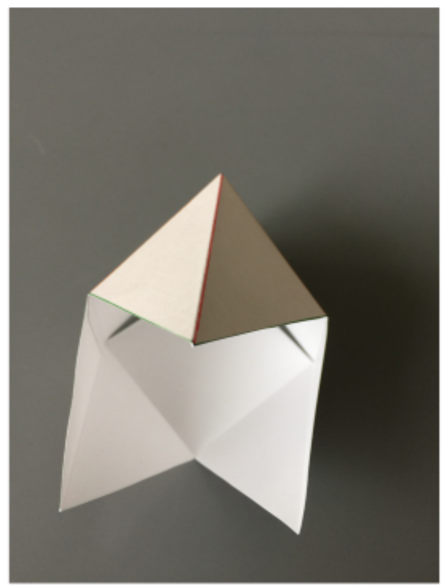
Th. 1 The above double flexatube is reversible. Moreover the folded layers do not sandwiched another layer.

Cube (Cutting face)





Cube (Cutting edge)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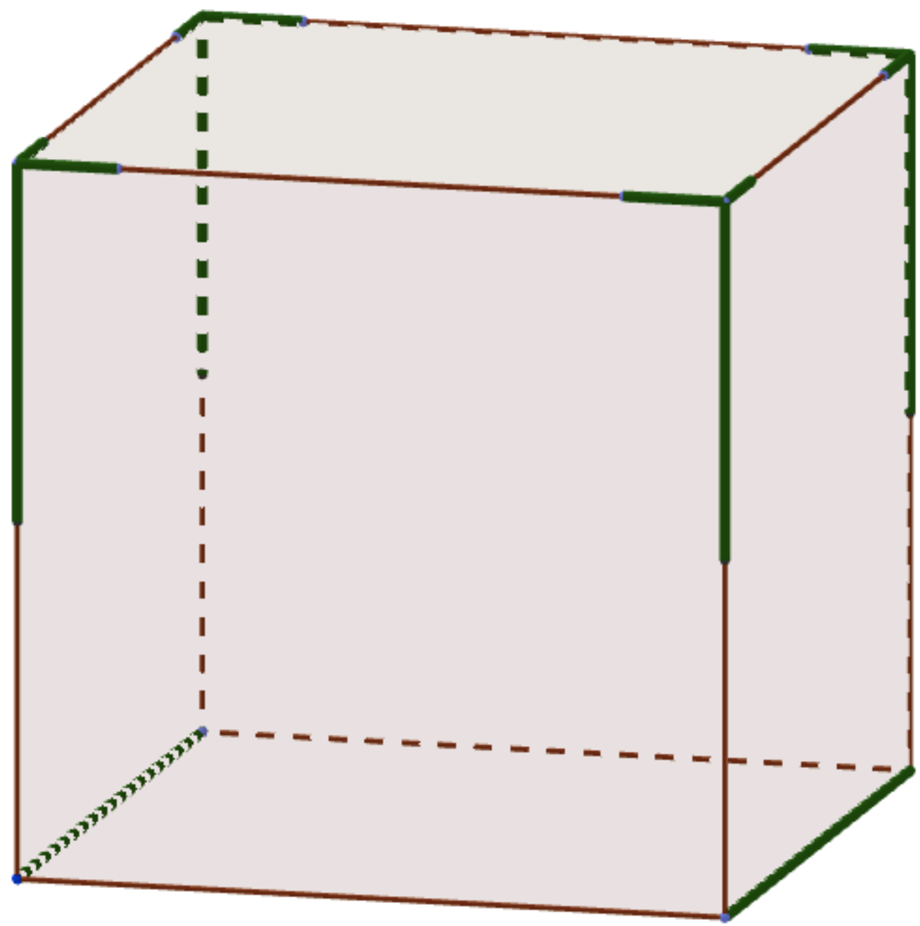
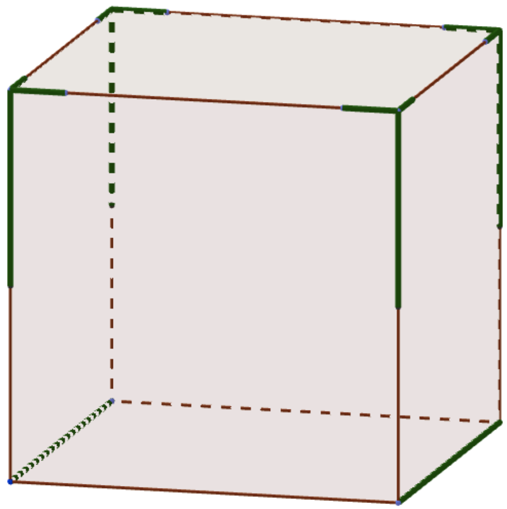
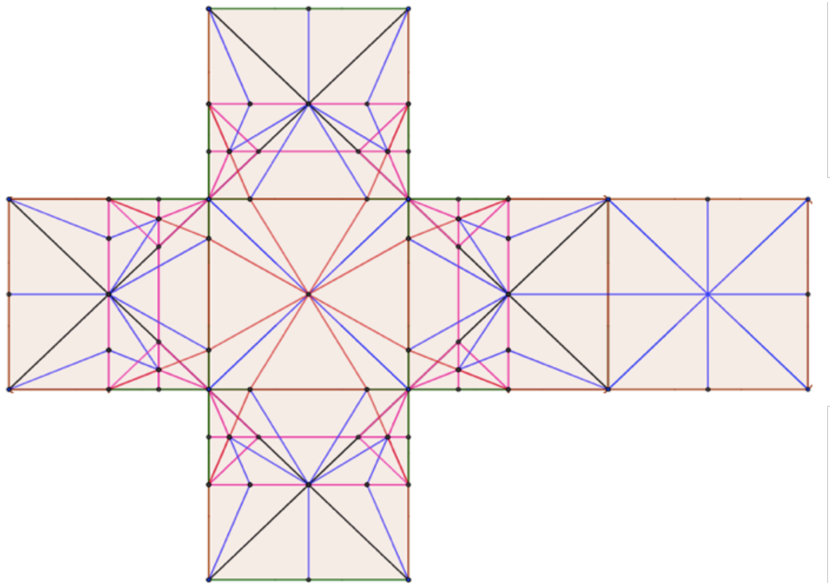
Cube with (green) slits (only on its edges)



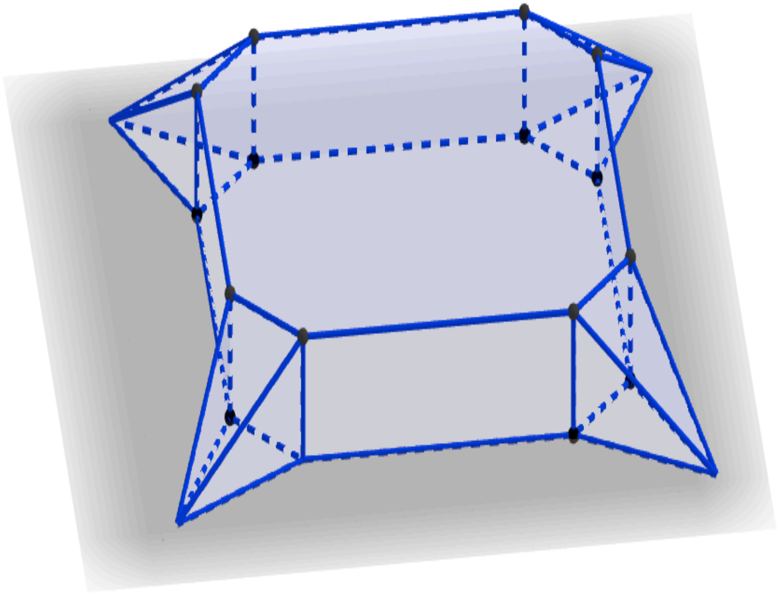
a cubical surface with slits whose length $4+\varepsilon$

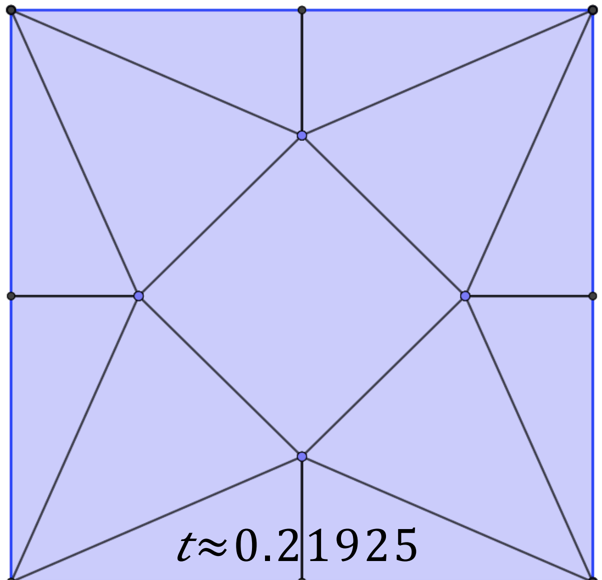




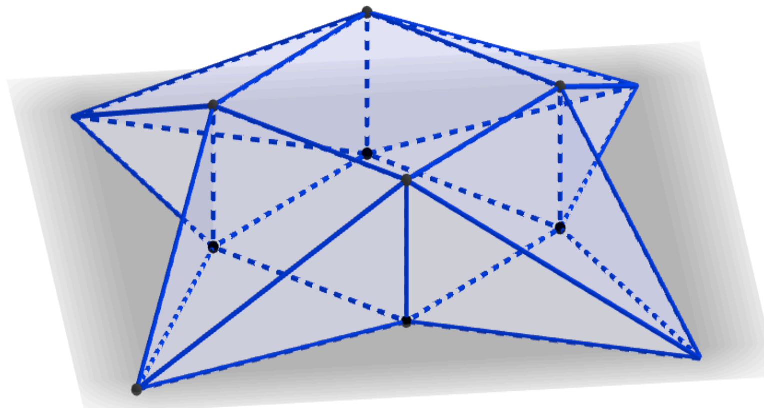
Th. 2 The above cubical surface with slits is reversible after adequate subdivisions with respect to doubleflexatube .

定義 : 正方形の紙をorigami-deformation により折ってその周の全てを平面に接させることができる立体を「折り紙テント」と呼ぶ.





$t \approx 0.21925$
5



$V \approx 0.0878266$