散逸モデルのステップフロー法解析と数値シミュレーション

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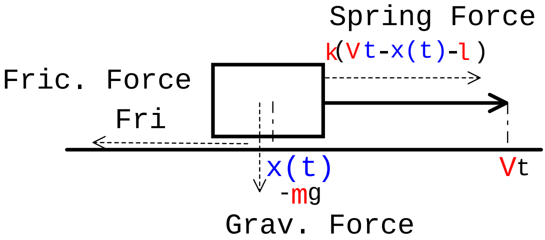
関連論文

Tribology International 93PA(2016)446, S. Ichinose
"Non-equilibrium Statistical Approach to Friction Models"

Sec 1. Introduction: a.Dissipative Model.

Figure: *1 The spring-block model, (7).*



Sec 1. Introduction: b.Dissipative Model.

Frictional Force

$$Fri \;=\; -\eta\dot{x}\ \text{(rain drop)},\quad -m\frac{\kappa\ \mathrm{sgn}(\dot{x})}{1+2\alpha|\dot{x}|}\ \text{(stick slip motion)},$$
$$m:\ \text{block mass } (M),\quad k:\ \text{spring const. } (MT^{-2}),$$
$$\text{frictional parameters}\ \ \alpha = 2.5\ \ (TL^{-1}),\ \ \kappa = 1.0\ \ (LT^{-2}),$$
$$\bar{\ell}:\ \text{block length } (L),\quad \bar{V}:\ \text{Velocity of spring top } (LT^{-1})\quad . \qquad (1)$$

1. Burridge and Knopoff, Bull. Seismol. Soc.Am.1967
2. Carlson and Langer PRL, PRA 1989
'Mechanical model of an earthquake fault'
3. Mori and Kawamura, J. Geoph. Res. 2006
'Simulation study of the one-dimensional Burridge-Knopoff model of earthquakes'

Sec 1. Introduction c.Burridge-Knopoff Model

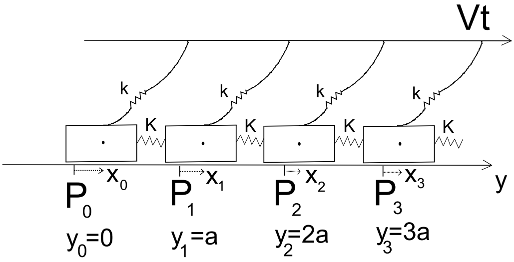
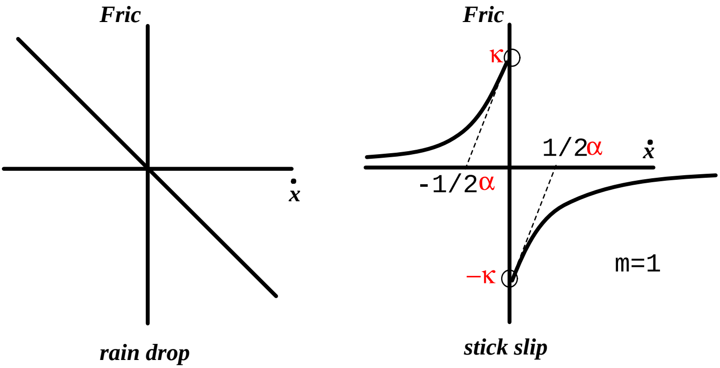


Figure: 2 *Burridge-Knopoff Model*

Sec 1. Introduction: d.Friction Forces.

Figure: 3 Friction Forces *[left] rain drop* *[right] stick slip model.*



Sec 1. Introduction: e.Energy with Dissipation

The classical equation of the dissipative block (Stick-Slip).

$$\ddot{x} + \frac{\kappa \ \mathrm{sgn}(\dot{x})}{1 + 2\alpha|\dot{x}|} + \omega^2 x = \omega^2(\bar{V}t - \bar{\ell}). \qquad \ddot{x} + \frac{\eta}{m}\dot{x} + \cdots \tag{2}$$

This has been solved numerically by Runge-Kutta method (Continuous Time Approach).
Energy conservation equation : Hamiltonian

$$H[\dot{x}, x] \equiv \frac{1}{2}\dot{x}^2 + \frac{\omega^2}{2}x^2 + \omega^2\bar{\ell}x + \underline{\int_0^t \frac{\kappa \ |\dot{x}|}{1 + 2\alpha|\dot{x}|} d\tilde{t}} - \underline{\omega^2 \bar{V} \int_0^t \tilde{t}\dot{x} d\tilde{t}}$$
$$= \left(\frac{1}{2}\dot{x}^2 + \frac{\omega^2}{2}x^2 + \omega^2\bar{\ell}x\right)\Big|_{t=0} = E_0 = -0.5 \ . \tag{3}$$

Three types of energy: 1 [4th] Dissipative energy (hysteresis); 2 [5th] External work (hysteresis); 3 Others. $\dot{x} = dx(\tilde{t})/d\tilde{t}$, $0 \leq \tilde{t} \leq t$.

Sec 1. Intro.: f.Discrete Morse Flow(DMF) Theory

- Time should be re-considered, when dissipation occurs.
  Non-Markovian effect, Hysteresis effect
  $\rightarrow$ Step-Wise approach to time-development.
- Connection between step n, $n-1$ and $n-2$ is determined by the minimal principle of an energy function.
- Time is "emergent" from the principle.
- Direction of flow (arrow of time) is built in from the beginning.
  $\mathrm{n} = 0 < 1 < 2 < \cdots$

New approach to Statistical Fluctuation
  Discrete Morse Flow Method(Kikuchi, '91)
  Holography (AdS/CFT, '98)

Sec 1. Introduction: g.Continuous vs DMF

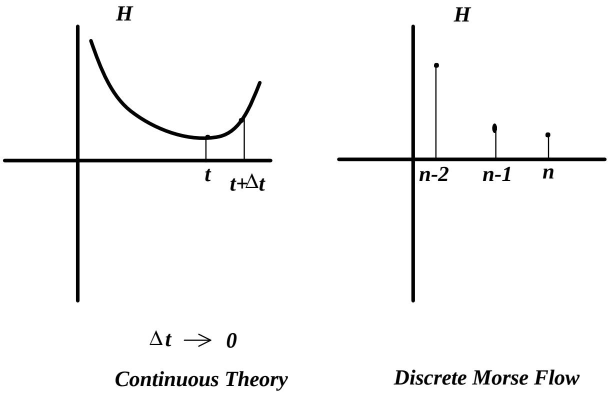


Figure: 4 *[left] Continuous Theory [right] Discrete Morse Flow.*

Sec 2. Spring-Block Model a. Discrete Morse Flow Theory

n-th Energy Function ('Action' of Recursion Rel. (5))

$$K_n(x) = V(x) - hnk\bar{V}x + m\frac{\kappa\ \mathrm{sgn}(x_{n-1} - x_{n-2})}{1 + 2\alpha|x_{n-1} - x_{n-2}|/h}\ x + \frac{m}{2h^2}(x - 2x_{n-1} + x_{n-2})^2\ ,\ \ V(x) = \frac{kx^2}{2} + k\bar{\ell}x, \qquad (4)$$

x: general position (L), x_{n-1}: $(n-1)$-th , x_{n-2}: $(n-2)$-th ,
h: 1 step interval (T)

$$K_n(x_n)\ \leq\ K_n(x_{n-1}),\ K_n(x_{n-2})$$

Sec 2. Spring-Block Model b.Variat. Principle

Minimal Energy Priciple $\delta K_n(x)/\delta x|_{x=x_n} = 0.$

$$\frac{k}{m}(x_n + \bar{\ell} - nh\bar{V}) + \frac{1}{h^2}(x_n - 2x_{n-1} + x_{n-2}) + \frac{\kappa\ \mathrm{sgn}(x_{n-1} - x_{n-2})}{1 + 2\alpha|x_{n-1} - x_{n-2}|/h} = 0\ ,\ \omega \equiv \sqrt{\frac{k}{m}}\ , \qquad (5)$$

where $n = 2, 3, 4, \cdots, N-1, N.$
Recursion relation among n-th, (n-1)-th and (n-2)-th positions

Parameters: $\bar{\ell} = 1,\ \bar{V} = 0.1,\ \omega = 1.0,\ \kappa = 1.0,\ \alpha = 2.5$
1 Step Interval: $h = 2.5 \times 10^{-3}$, Total Step Number: $N = 2 \times 10^4$
($h \cdot N = 50$ Total Step Length('Time'))
Initial condition: $x_0 = -\bar{\ell},\ (x_1 - x_0)/h = 0.$
See 3.a,b Movement, 3.c,d Velocity

Sec 2. Spring-Block Model c.Continuous Limit

Continuous Limit

$$h \to 0\ ,\ \ n \to \text{large}\ ,\ \ nh = t_n \to t\ ,$$
$$\frac{x_{n-1} - x_{n-2}}{h} \to \dot{x}\ ,\ \frac{x_n - 2x_{n-1} + x_{n-2}}{h^2} \to \ddot{x}\ , \tag{6}$$

$$m\ddot{x} = k(\bar{V}t - x - \bar{\ell}) - m\frac{\kappa\ \text{sgn}(\dot{x})}{1 + 2\alpha|\dot{x}|} \quad . \tag{7}$$

This is the spring-block model. See Fig.1.

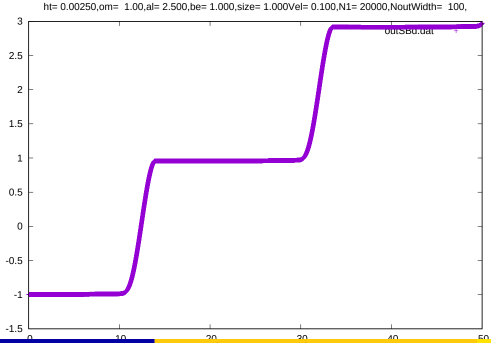
♣ DMF Numerical Result

The graph of movement (x_n, eq.(5)) is shown in Fig.5.

♣ CT approach: Solving (7) numerically by Runge-Kutta 4. Same movement is obtained.

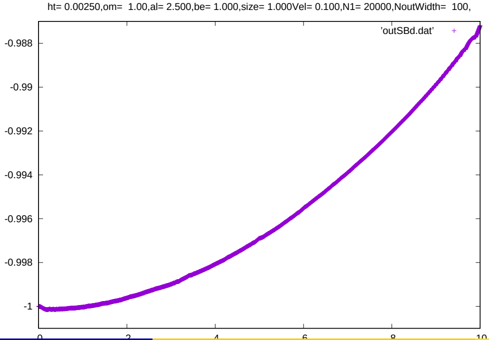
Sec 3. a.Movement, DMF

Figure: 5 SB Model, Movement, x_n *The step-wise solution (5) correctly reproduces the continuous-time solution:* *stick-slip motion* $O(10^{-1})$



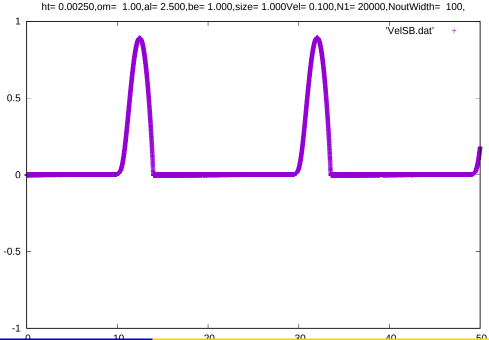
Sec 3. b.Precise Movement, DMF

Figure: 6 Spring-Block Model, Precise Movement, x_n Precise plot for the step-interval $0 \le t \le 10$. Very-slowly-increasing curve. $O(10^{-4})$



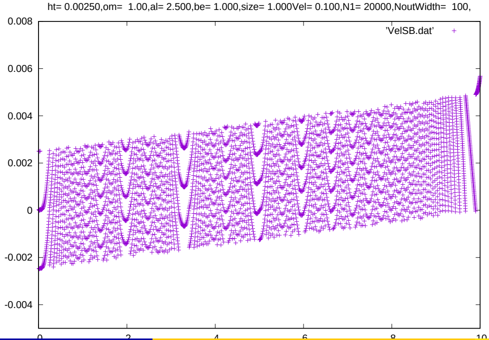
Sec 3. c.Velocity, DMF

Figure: 7 Spring-Block Model, Velocity, $(x_n - x_{n-1})/h$ The step-wise solution (5) correctly reproduces the continuous-time solution. $O(10^{-1})$



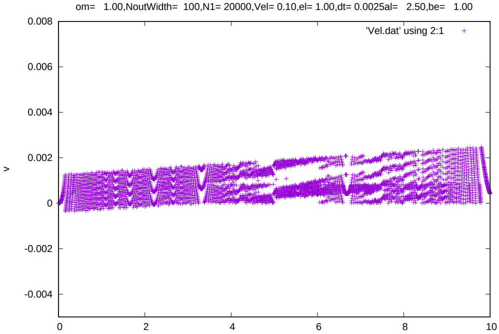
Sec 3. d.Precise Velocity, DMF

Figure: 8 SB Model, Precise Velocity, $(x_n - x_{n-1})/h$ The step-interval $0 \le t \le 10$. CT-approach gives different fluctuation graph. $O(10^{-3})$



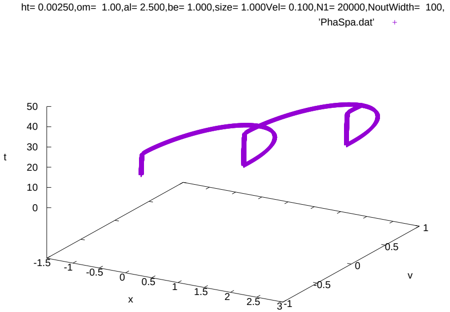
Sec 3. e.Precise Velocity, CT

Figure: 9 SB Model, Precise Velocity, $(x_n - x_{n-1})/h$ The step-interval $0 \leq t \leq 10$. CT-approach. $O(10^{-3})$



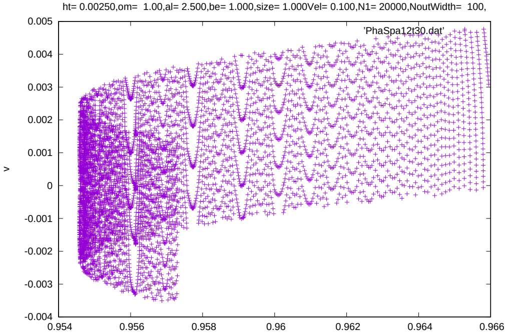
Sec 4. a.Phase Space, DMF

Figure: 10 SB Model, Phase Space, (x, v, t) Stereo graphic plot.
$0 \leq t \leq 50$



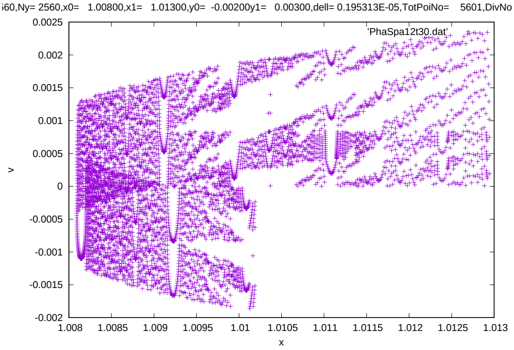
Sec 4. b.Phase Space $(x(t), v(t))$ for $15 \leq t \leq 29$

Figure: 11 SB Model, Phase Space, $(x, v,)$ plot for $15 \leq t \leq 29$. Second stick region.



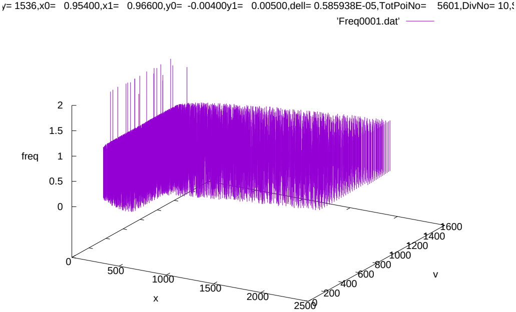
Sec 4. c.Phase Space for $15 \leq t \leq 29$, CT

Figure: 12 SB Model, Phase Space, $(x, v,)$ plot for $15 \leq t \leq 29$.



Sec 4. d.Phase Space 'Frequency' for $15 \leq t \leq 29$

Figure: 13 SB Model, Frequency plot for $15 \leq t \leq 29$. (x, v) is represented by the 'box-numbder' (n_x, n_y).



Sec 4. e.Fractal Dimension

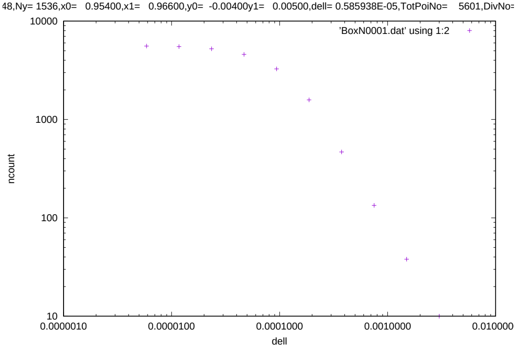
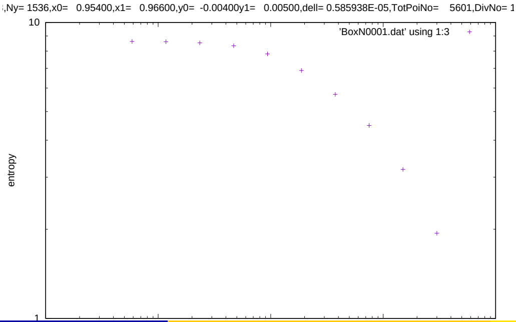


Figure: 14 Box size vs No of boxes the graph has. Log-Log plot.

Sec 4. f.Entropy for Different Sizes

Figure: 15 Shannon entropy $S = -\sum_k^K p_k \ln p_k$. For a fixed size, 'k' runs for all non-empty boxes. p_k = No of frequency in k-th box/ total frequency numbers.



Sec 5. a. Three Types of Energy

Three types of energy

$$\text{DMF-energy} \quad \mathcal{E}_n \equiv K_n(x_n)/m = MAR_n + EXT_n + DIS_n \quad ,$$

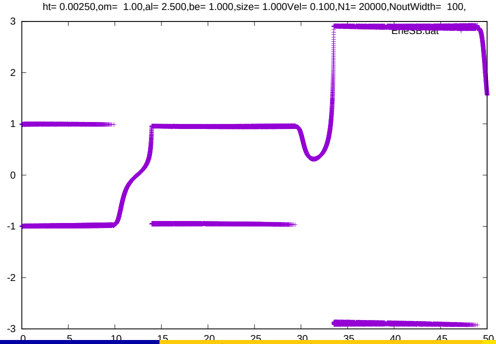
1. *DissipativeEnergy* : $DIS_n = \dfrac{\kappa \ \mathrm{sgn}(x_{n-1} - x_{n-2})}{1 + 2\alpha |x_{n-1} - x_{n-2}|/h} x_n$,
2. *ExternalEnergy* : $EXT_n = -hn\omega^2 \bar{V} x_n$,
3. *MarkovianEnergy* :

$$MAR_n = \frac{1}{m} V(x_n) + \frac{1}{2h^2}(x_n - 2x_{n-1} + x_{n-2})^2 \quad . \qquad (8)$$

1 and 2 do not have hysteresis effect.

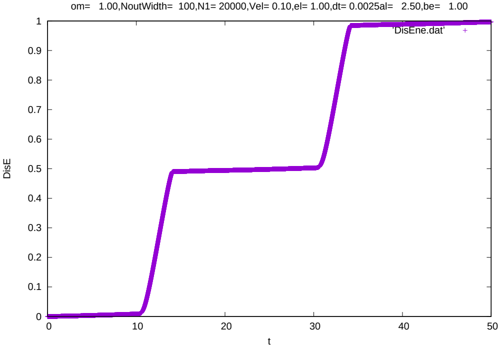
Sec 5. b.Dissipative Energy, DMF

Figure: 16 Diss. Ene., DMF DIS_n of (8). Stick int.: 2 energy splitting $\pm\epsilon$. ϵ is 'quantized'. Slip int.: connect $-\epsilon$ of a stick region to $+\epsilon'$ of next one.



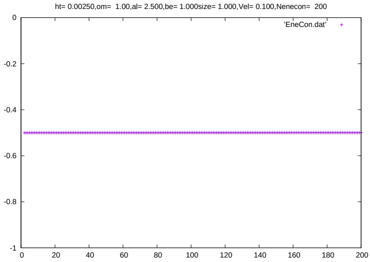
Sec 5. c.Dissip. Energy, CT-approach

Figure: 17 Dissipative Energy, CT-approach, $\int_0^t \kappa \ |\dot{x}|/(1+2\alpha|\dot{x}|)d\tilde{t}$ of (3).



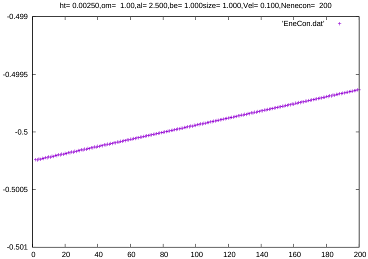
Sec 5. d.Energy Conservation H_n, DMF

Figure: 18 *Energy Conservation, DMF* , $h = 0.0025$, $N = 200$, The energy conservation is valid in the order of 10^{-1}. H_n is the discretized $H[\dot{x}, x]$ in CT.



Sec 5. e.Energy Conservation H_n, DMF

Figure: 19 *Energy Conservation, DMF* , $h = 0.0025$, $N = 200$, The n=0 value, -0.5001, is different from $E_0 = -0.5$. 10^{-4}



Sec 5. f. Conserved Energy

Corrected energy

$$
\begin{aligned}
H_n^R = H_n - \gamma \cdot nh \quad , \\
1.\ \kappa = 0.5 :\ \gamma = 0.316 \times 10^{-3} \\
2.\ \kappa = 1.0(displayed) :\ \gamma = 1.21 \times 10^{-3} \\
3.\ \kappa = 2.0 :\ \gamma = 4.92 \times 10^{-3}
\end{aligned}
\tag{9}
$$

Roughly $\gamma \propto \kappa^2$.

Sec 6. Frictional Energy a.Definition

Minimal Energy Priciple $\delta K_n(x)/\delta x|_{x=x_n} = 0.$

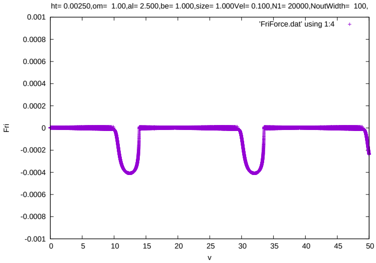
$$\frac{k}{m}(x_n + \bar{\ell} - nh\bar{V}) + \frac{1}{h^2}(x_n - 2x_{n-1} + x_{n-2}) + \frac{\kappa \operatorname{sgn}(x_{n-1} - x_{n-2})}{1 + 2\alpha|x_{n-1} - x_{n-2}|/h} = 0 \ , \ \omega \equiv \sqrt{\frac{k}{m}} \ , \tag{10}$$

where $n = 2, 3, 4, \cdots, N-1, N.$

$$Fri_n = -\frac{\kappa \operatorname{sgn}(x_{n-1} - x_{n-2})}{1 + 2\alpha|x_{n-1} - x_{n-2}|/h}, \ \ FriE(n) = Fri_n \times (x_n - x_{n-1}) \quad .(11)$$

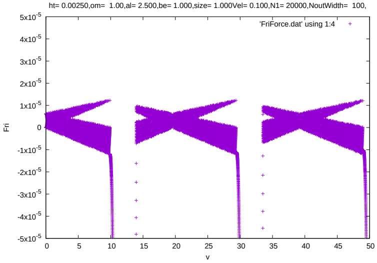
Sec 6. Frictional Energy b.Simulation Result

Figure: 20 *Friction Energy* , Scale 10^{-4}. Horizontal axis is 'time' t.



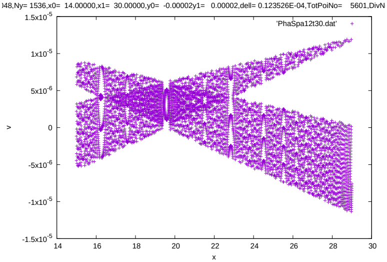
Sec 6. Frictional Energy c.Precise Result

Figure: 21 *Precise Friction Energy* , Scale 10^{-6}. Horizontal axis is 'time' t.



Sec 6. Frictional Energy d. $15 \leq t \leq 29$

Figure: 22 *Precise Friction Energy* , Scale 10^{-6}. Second stick region. Horizontal axis is 'time' t.



Sec 6. Frictional Energy e.Frequency

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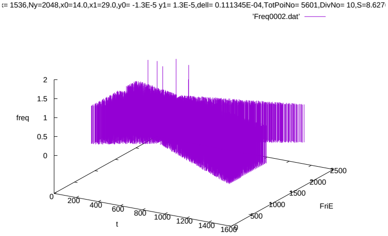
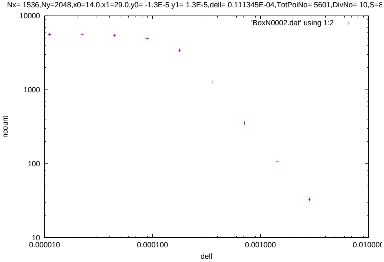
Figure: 23 Plot of (t, FriE, frequency). The plane (t, FriE) is represented by 'box-numbers' (n_x, n_y)



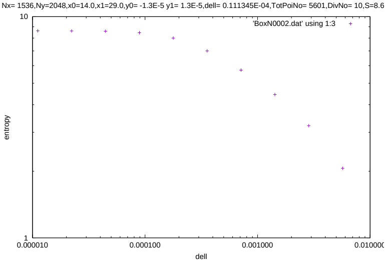
Sec 6. Frictional Energy f.Fractal Distribution

Figure: 24 Plot of (box-size, frequency). Log-Log plot.



Sec 6. Frictional Energy f.Entropy versus box-size

Figure: 25 Entropy versus box-size. Log-Log plot.



Sec 7. Geom. of Dissip. Sys. a. Bulk(t, X, P) Metric

Spring-Block Model (rain drop)

$$\Delta {s_n}^2 = 2\, dt^2 V_1(X_n) + (\Delta X_n)^2 + (\Delta P_n)^2,$$
$$V_1(X_n) \equiv \frac{k}{2}\{(\frac{X_n}{\sqrt{\eta h}})^2 + 2\bar{\ell}\frac{X_n}{\sqrt{\eta h}} - 2nh\bar{V}\frac{X_n}{\sqrt{\eta h}}\}, \quad (12)$$

where $X_n \equiv \sqrt{\eta h} x_n,\ P_n/\sqrt{m} \equiv hv_n = (x_n - x_{n-1}),$.

Spring-Block Model (stick-slip)

$$\Delta {s_n}^2 = dt^2\omega^2\{{X_n}^2 + 2(\bar{\ell} - nh\bar{V})X_n\} \pm \frac{2(dt)^2\kappa}{1+2\alpha|P_n|}X_n + (\Delta P_n)^2 \quad .(13)$$

Sec 7. Geom. of Dissip. Sys. b.Statistical Models

Spring-Block Model (rain drop)

$$\mathrm{e}^{-\beta\alpha^{-1}F} = \int \prod_n dy_n dw_n \mathrm{e}^{-\frac{1}{\alpha}L_S},$$

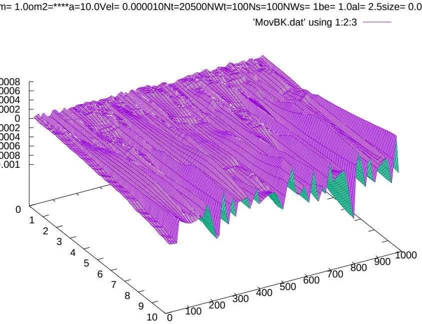
$$L_S = \int_0^\beta ds|_{on-path} = h\sum_{n=0}^{\beta/h}(2V_1(y_n) + \dot{y}_n^2 + \dot{w}_n^2), \quad (14)$$

Spring-Block Model (stick-slip)

$$L_S = \int_0^\beta ds|_{on-path} = h\sum_{n=0}^{\beta/h}\{\omega^2 {y_n}^2 + 2\omega^2(\bar{\ell} - \bar{V}nh)y_n \pm \frac{2\kappa}{1+2\alpha|w_n|}y_n + \dot{w}_n^2\}, \quad (15)$$

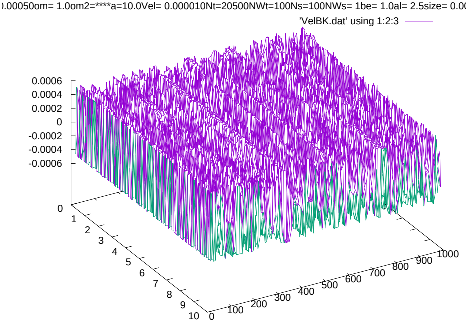
Sec 8. a.l10MovBK

Figure: 27 *Movement , DMF* Position $x_{n,i}$



Sec 8. b.l10VelBK

Figure: 28 *Velocity , DMF* Velocity $v_{n,i} = (x_{n,i} - x_{n-1,i})/h$.



Sec 9. Conclusion a. DMF vs Runge-Kutta

The different results between DMF and Runge-Kutta imply the importance to reexamine the statistical analysis of the Burridge-Knopoff model.

1. Slides Sec.3d, 3e : Velocity
2. Slides Sec.4b, 4c: Phase Space
3. Slide Sec.5b, 5c: Dissipative Energy

Sec 9. Conclusion b.Finally

The dissipative systems are solved by the minimal principle. The difficulty of the hysteresis effect (non-Markovian effect) is avoided in the present approach. These are the advantage of the discrete Morse flow method. We do not use the ordinary time t, instead, exploit the step number n ($t_n = nh$). Several theoretical proposals for the statistical ensembles, appearing in the friction phenomena, are made. It is necessary to *numerically* evaluate the models with the proposed ensembles and compare the result with the real data appearing both in the natural phenomena and in the laboratory experiment.