やりたいこと Dec. 2024

[KO] 13/18 Thorem 1.

Then there is a (2k-2)-form W on P such that

$$Tf(\omega') - Tf(\omega) = dW.$$

を、[CM] 1/53

Our basic assumption on M is ,that it be:nondegenerate,

or

[T76] 47/60

A $\tilde{G}$-structure (P, B) of type $\mathfrak{M}$ on a manifold M is said to be integrable if the corresponding almost PC structure (D, I) is integrable.

THEOREM 11.1. Let (P, ω) be a normal connection of type $\mathfrak{G}$ on a manifold M, and $(\tilde{P}, \tilde{\theta})$ the corresponding $\tilde{G}$-structure of type $\mathfrak{M}$. Then the $\tilde{G}$-structure $(\tilde{P}, \tilde{\theta})$ is integrable if and only if the g_{-1}-component K_{-1} of the curvature K (of (P, ω)) vanishes.

[CM] 33/53

We consider the real line bundle E, which consists of the multiples $u\theta, u(> 0)$ being a fiber coordinate.

[CM] 34/53

Denote by Y its principal G_1-bundle. Then we have

$$G_1 \xrightarrow{f} Y \xrightarrow{\pi} E, \tag{4.14}$$

46/53

THEOREM 5.1. Given a non-degenerate integrable G-structure on a manifold M of dimension $2n + l$. Consider the principal bundle

Y over E with the group $G_1 \subset SU(p+1, q+1)/K$. There is in Y a uniquely defined connexion with the group $SU(p+1, q+1)$, which is characterized by the vanishing of the torsion form and the condition (5.39).

[BDS] 13/25

In [CM], Chern associates to such an M, funetorially for local CR-equiwlenees, a bundle of coframes Y^{n^2+2n} over $E^{2n} = Y/H_0$, which is an H-bundle over M, with (funetorial) Cartan connection on Y with values in $\mathfrak{su}(p+1, q+1)$, and fiber group H.

[BDS] 14/25

$$Y/H_1 = \widehat{E} \qquad\qquad E = Y/H \tag{4.4}$$

[T76] 27/60

Now let M be a manifold of dimension $2n-1$ and let P be a principal fibre bundle over the base space M with structure group G'. Let θ be an $\mathfrak{m}$-valued 1-form on P and let α be an $\mathbb{R}\cdot E$-valued 1-form on P. Then we say that the collection $\{P, \theta, \alpha\}$ is an E-system on M if it satisfies the following conditions:

(E.1) Let X be a tangent vector to P. Then $\theta(X) = 0$ if and only if X is a vertical vector in the principal bundle P;

(E.2) $R_a^*\theta = \rho(a)^{-1}$ for all $a \in G'$;

(E.3) $R_\alpha \alpha = \alpha + (\mathrm{Ad(a^{-1})}\theta)_{\mathrm{E}}$ for all $\alpha \in G'$;

(E.4) $\alpha(X^*) = X_E$ for all $X \in \mathfrak{g}'$;

(E.5) $d\theta_{-2} + 1/2 \cdot [\theta_{-1}, \theta_{-1}] + [\alpha, \theta_{-2}] = 0$, where θ_p is the $\mathfrak{g}_p$-component of θ with respect to the decomposition: $\mathfrak{m} = \sum_{p<0} \mathfrak{g}_p$.

Note that (E.1) and (E.2) are the very conditions satisfied by the basic form θ for a connection ω of type $\mathfrak{G}$ in P (see Lemma 1.1). Therefore we know from the argument above that if (P, ω) is a connection of type $\mathfrak{G}$ with. $K_{-2} = 0$, then the collection $\{P, \omega_{-2} + \omega_{-1}, \omega_E\}$ gives an E-system.

Just as in the case of a general Cartan connection (see 1.2), every E-system $\{P, \theta, \alpha\}$ on a manifold M induces a $\tilde{G}$-structure $(\tilde{P}, \tilde{\theta})$ on M in a natural way. From (E.5) it is clear that the $\tilde{G}$-structure is of type $\mathfrak{M}$ (cf.Lemma 4.3).

[T76] 29/60

PROPOSITION 7.2. Assume that $\mathfrak{h}_0^{(1)} = \{0\}$. Let $\{P, \theta, \alpha\}$ be an E-system on a manifold M. Then there is a unique normal connection ω of type $\mathfrak{G}$ in P compatible with θ and α, i.e., $\theta = \omega_{-2} + \omega_{-1}$ and $a = \omega_E$.

Theorem 4.4 follows immediately from Proposition 7.1 and 7.2.

[T76] 33/60

LEMMA 8.2 (Bianchi's identity). For all $X, Y, Z \in \mathfrak{m}$ we have:

$$\mathfrak{S}[K(X,Y), Z] - \mathfrak{S}(\breve{\omega}(X)K)(Y,Z) + \mathfrak{S}K([X,Y],Z) - \mathfrak{S}K(K_{-1}(X,Y),Z) = 0,$$

where $\mathfrak{S}$ stands for the cyclic sum with respect to X, Y, Z.

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