Corrections to :
A. Hora, N. Obata, Quantum Probability and Spectral Analysis of Graphs, TMP, Springer, 2007

Chapter 11

• In p.300 the text *from Equation (11.3) until the bottom of that page* is subject to the following replacement. (Equation (11.3) is incorrect. M_{k+1} is to be replaced by free cumulant R_{k+1}, while we do not use this in the following corrected text.) After the sentence containing Equation (11.2), we proceed as follows.

Then $M_k(\mathfrak{m}_\lambda)$ is expressed as a polynomial in $\Sigma_{k-1}(\lambda), \Sigma_{k-2}(\lambda), \cdots$ in which each term has at most weight degree k. See Sect. 11.6 for details.

We consider a family of (11.1) indexed by $k = 1, 2, \ldots$. Since the kth moment of a $1/\sqrt{n}$-rescaled diagram gets $n^{-k/2}$-multiple, the right hand side of (11.1) is equivalently replaced by

$$n^{-(k+2)/2} P(\Sigma_{k+1}, \Sigma_k, \cdots) - (\text{constant}), \tag{11.3}$$

where a term in the polynomial $P(\Sigma_{k+1}, \Sigma_k, \cdots)$, say

$$\Sigma_{k_1} \Sigma_{k_2} \cdots \Sigma_{k_l} \Sigma_1^r, \qquad k_i \geq 2, \ r \geq 0,$$

has weight degree $(k_1 + 1) + \cdots + (k_l + 1) + 2r \leq k + 2$. On the other hand, Kerov's central limit theorem for irreducible characters of $S(n)$ below ensures that Σ_j's behave as independent $n^{j/2}$-multiple Gaussian random variables asymptotically. Hence the contributing order of $\Sigma_{k_1} \cdots \Sigma_{k_l} \Sigma_1^r$ in (11.3) is

$$-\frac{k+2}{2} + \frac{k_1}{2} + \cdots + \frac{k_l}{2} + r \leq -\frac{l}{2}.$$

Note that the constant term in (11.3) vanishes asymptotically (together with the $l = 0$ term) since we already know that (11.3) tends to 0 as $n \to \infty$. The dominant terms are thus those of $l = 1$. Then, rescaled by $\sqrt{n}$-multiple, (11.3) converges to a sum of Gaussian random variables.
