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Methods in Applied Mathematics I, 2007

Report Problems

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▶ Write your name, affiliation and year on the top page.
▶ Submit your report by **February 1 (Fri)** into **the box in the control room**. (Choose the correct box because there are several boxes for report submissions.)
▶ Your report should be written in English.

Problem 1. (1) Let $(\Omega, \mathcal{F}, P)$ be a probability space. Show that

$$P\Big(\bigcap_{n=1}^{\infty} A_n\Big) = \lim_{n\to\infty} P(A_n)$$

holds if $A_1, A_2, \cdots \in \mathcal{F}$ and $A_1 \supset A_2 \supset \cdots$ (i.e. A_n's are decreasing events).

(2) In our coin tossing model, show that $P(\{1, 1, 1, \cdots\}) = 0$ holds.

Problem 2. Consider infinite trials of coin tossing, HEAD comig out with probability p at each trial. Let X be the number of HEADs before k TAILs appear.

(1) Find the probability $P(X = n)$ for each $n = 0, 1, 2, \cdots$.

(2) This is called the Pascal distribution or a negative binomial distribution. Why is it referred to as "negative binomial"?

Problem3. Do something about the gamma function:

$$\Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} dt, \qquad x > 0$$

(especially on asymptotic behavior as $x \to \infty$).

Problem 4. (1) Check properties of the rate function

$$f(x) = (p + x) \log\big(1 + \frac{x}{p}\big) + (1 - p - x) \log\big(1 - \frac{x}{1-p}\big), \qquad -p \le x \le 1 - p$$

to draw a figure of the graph of $y = f(x)$.

(2) For positive sequences (a_n) and (b_n), show that

$$\lim_{n\to\infty} \frac{\log a_n}{n} = a \text{ and } \lim_{n\to\infty} \frac{\log b_n}{n} = b \quad \text{imply} \quad \lim_{n\to\infty} \frac{\log(a_n + b_n)}{n} = \max(a, b).$$