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CENTERS OF AFFINE HYPERSURFACES

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It is a classical result that proper affine hyperspheres are characterized as follows. Let $f : M \to \mathbb{R}^{n+1}$ be an affine immersion with Blaschke normal vector field ξ. The affine support function ρ from the origin o is defined as

$$f(x) = f_* Z_x + \rho(x)\xi_x,$$

where Z_x is in $T_x M$. Then f is an affine hypersphere with center o if and only if the function ρ is constant, which is called the radius. Moreover, it is equivalent to $f - \rho\xi : M \to \mathbb{R}^{n+1}$ is the constant map o.

Definition. For an affine hypersurface f, we call $f - \rho\xi$ the center map of f, and denote it by c_f.

For example, the center map of Wang's centroaffine minimal surface $f_{ab}(u,v) := {}^t(u, v, u^{-a}v^{-b})$, where a, b are positive constants, is calculated as

$$c_{f_{ab}}(u,v) = \begin{bmatrix} -\frac{1-2a+b}{2a} & 0 & 0 \\ 0 & -\frac{1+a-2b}{2b} & 0 \\ 0 & 0 & -\frac{-2+a+b}{2} \end{bmatrix} f_{ab}(u,v).$$

In the case that $a = b = 1$, f_{ab} is an affine sphere and $c_{f_{ab}}$ is the constant map o. We remark that the center map is centroaffinely congruent to the original immersion.

The problem is what the class of affine hypersurfaces whose center maps are centroaffinely congruent to themselves is, and whether this property characterizes an already-known class geometrically. This investigation is under way, and any comments you wish to make are most welcome.

Theorem. Let f be an affine hypersurface with center map c_f. If c_f is an immersion centroaffinely congruent to f, then the Tchebychev operator $\mathcal{T}$ vanishes identically.

In fact, we obtain that

$$h(X, \mathcal{T}(Y)) = -\frac{1}{2} h(\nabla_X Y - \check{\nabla}_X Y, T),$$

where h, ∇ and T are the centroaffine metric and connection, and the Tchebychev vector field induced by f, respectively, and $\check{\nabla}$ is the connection centroaffinely induced by c_f.

References

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