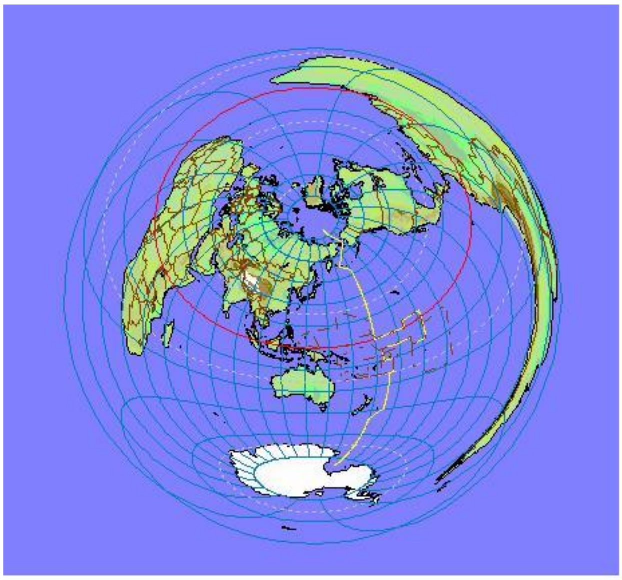
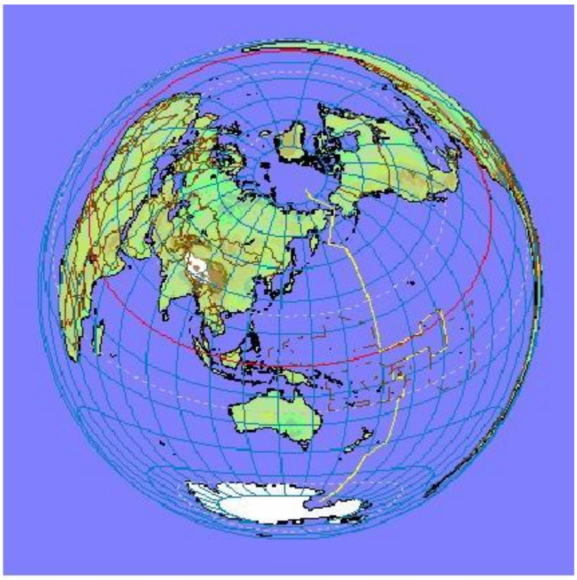
3 Bayesian statistics





一様性は座標系の取り方に依存する

左図：正距方位図法　　　　右図：正積方位図法

4 Tsallis statistics

Definition 4.1
$p(x;\mu,\sigma)$: *the* q**-normal distribution**

$$p(x;\mu,\sigma) = \frac{1}{Z_q}\left[1 - \frac{1-q}{3-q}\frac{(x-\mu)^2}{\sigma^2}\right]_+^{\frac{1}{1-q}}$$

$$\textit{where} \quad Z_q = \begin{cases} \frac{\sqrt{3-q}}{\sqrt{1-q}}\mathrm{Beta}\left(\frac{2-q}{1-q}, \frac{1}{2}\right)\sigma & (-\infty < q < 1) \\ \frac{\sqrt{3-q}}{\sqrt{1-q}}\mathrm{Beta}\left(\frac{3-q}{2(1-q)}, \frac{1}{2}\right)\sigma & (1 \leq q < 3) \end{cases}$$

$$[*]_+ = \max\{0, *\}$$

$q \to 1 \quad \Longrightarrow \quad p(x;\mu,\sigma)$ converges to normal distribution

$$p(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Remark 4.2
Normal distributions $\Longleftarrow$ the maximization of the Boltzmann-Gibbs-Shannon entropy.
q-Normal distributions $\Longleftarrow$ the maximization of the Tsallis non-extensive entropy.

$$ST_q(p) = \frac{1}{1-q}\left(\int_{-\infty}^{\infty}(p(x))^q dx - 1\right)$$

$$p(x;\mu,\sigma) = \frac{1}{Z_q}\left[1 - \frac{1-q}{3-q}\frac{(x-\mu)^2}{\sigma^2}\right]_+^{\frac{1}{1-q}}$$

q	distribution
$-\infty$	uniform on $[\mu-\sigma, \mu+\sigma]$
-1	semi-circle
1	normal
$1+\dfrac{1}{n+1}$	student t
2	Cauchy distribution
3	uniform on $[-\infty, \infty]$