粗い境界付近の粘性流体の数学解析

An Introduction to

Gérard-Varet, D., Masmoudi, N.: Relevance of the slip condition for fluid flows near an irregular boundary. Comm. Math.Phys. **295** (1), 99-137(2010)

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渦の特徴付け

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Viscous Fluid and No-slip Condition

・Incompressible fluid: fluid which does not change its volume (ex. water)
In this presentation, we deal with only incompressible fluids.

・Viscosity: fluid's resistance to flow

thin  thick

http://www.synlube.com/viscosit.htm

The motion of viscous fluid is described by the Navier-Stokes equations.

The viscosity of water is low (1.0×10^{-6} m^2/s at 20℃).

However, the effect of the viscosity can not be neglected near surfaces.

Let $\vec{u}$ (t,x) = (u_1(t,x), u_2(t,x), u_3(t,x)) be the velocity field of the fluid at time t and position x.

Then, $\vec{u}$ satisfies on the surface (=boundary)

$$\vec{u}\,(t,x) = (0, 0, 0).$$

This condition is called the no-slip boundary condition.

⇒ https://www.youtube.com/watch?v=cUTkqZeiMow

Namely, a viscous fluid “sticks” to the boundary.

Roughness-induced Effect

Flow near complex boundaries ?

(ex. geophysical fluid dynamics)

http://images.nationalgeographic.com/wpf/media-live/photos/000/911/cache/91190_990x742-cb1438273462.jpg

The structure of flows near a solid wall with a rough surface ?

Journal of Computational Physics **147**, 187-218 (1998)

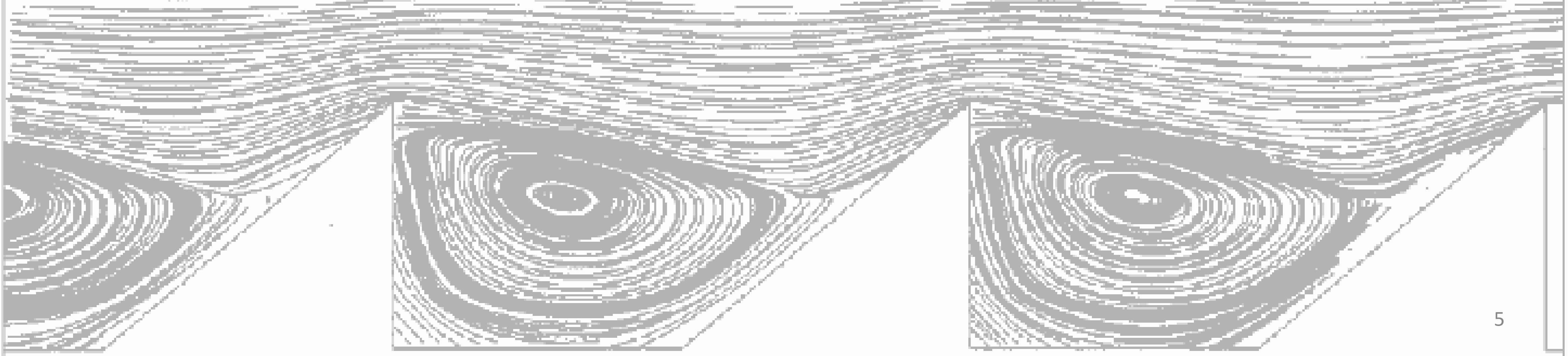
Achdou･Pironneau･Valentin (1998) : numerical analysis

Observation

Step1. Fluids flow into cavities with causing small-scale vortices

Step2. There is the large-scale flow away from the boundary

Step3. Near the boundary, the slipping effect is induced by the interaction between these two scales flows



Journal of Computational Physics **147**, 187-218 (1998)

It is hard to analyze Step1 - Step3 mathematically.

There are still many unknown aspects of the creation of vortices near boundaries.

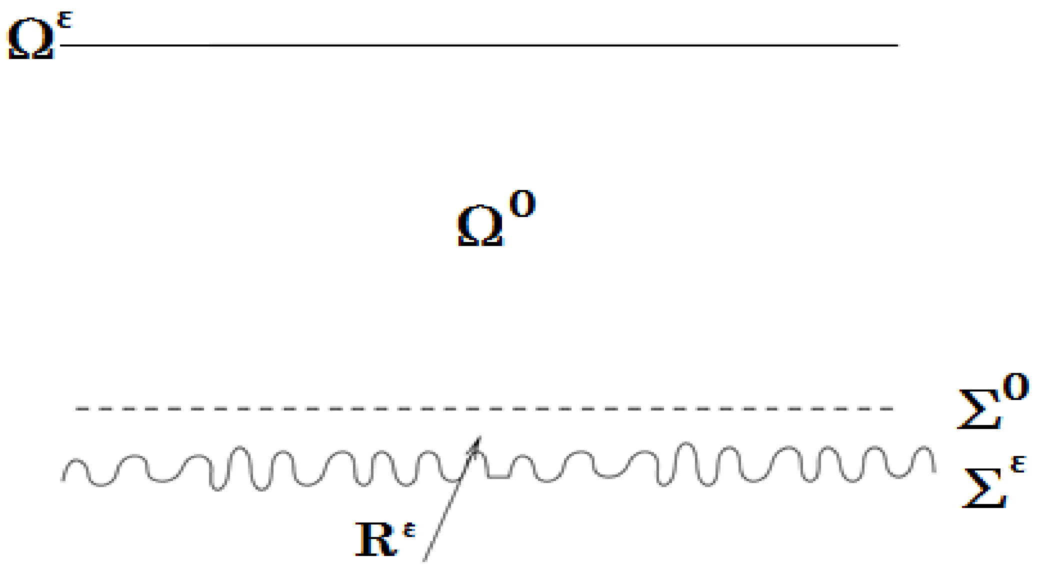
The slip induced by the boundary-roughness is analyzed by

Gérard-Varet, D., Masmoudi, N.: Relevance of the slip condition for fluid flows near an irregular boundary. Comm. Math. Phys. **295** (1), 99-137(2010).

⇒ Averaged effect of roughness

Model

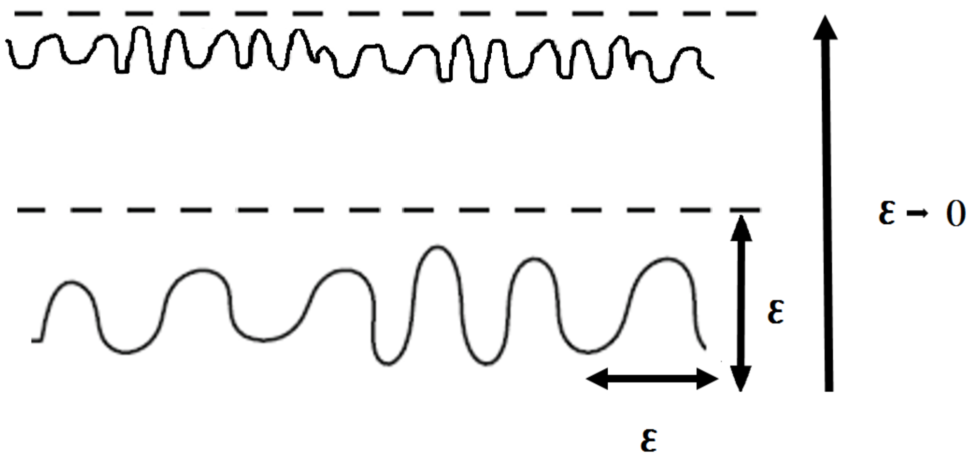
- Ω^ε : two-dimensional channel with a periodic rough boundary Σ^ε
- $\vec{u}^\varepsilon = (u_1^\varepsilon(x), u_2^\varepsilon(x))$: stationary flow in Ω^ε, flowing from left to right, viscosity = 1



The amplitude and the pulse width of Σ^ε are assumed to be of order ε.

When ε→0,

1. Σ^ε oscillates highly (gets rougher)
2. Σ^ε gets closer to some flat boundary Σ^0



Problem: Profile of $\overrightarrow{u^{\varepsilon}}$ when $\varepsilon \to 0$?

If "roughness induces slip" is correct, a precise profile of $\overrightarrow{u^{\varepsilon}}$ should be given by a flow slipping on Σ^0.

We consider the two flows which are expected to give a approximation of $\overrightarrow{u^{\varepsilon}}$.

1. Flow $\vec{u}^0$ which does not slip on Σ^0

Idea Since $\vec{u}^\varepsilon = (0, 0)$ on Σ^ε,

approximately $\vec{u}^\varepsilon = (0, 0)$ on Σ^0.

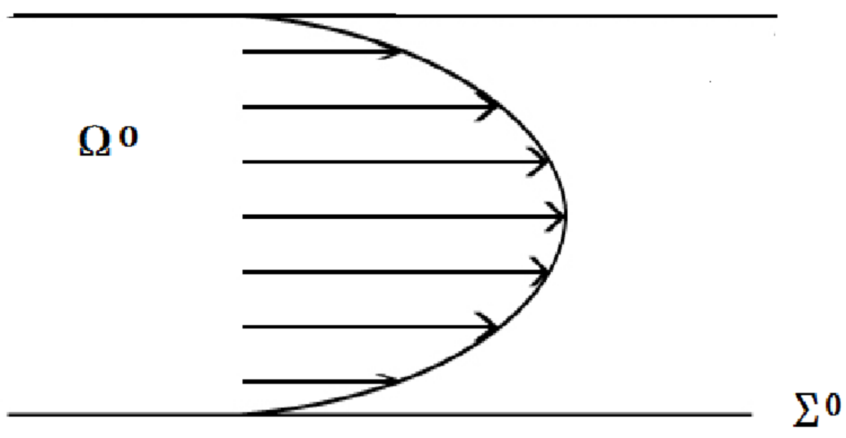


figure of $\vec{u}^0$

2. Flow $\vec{u}_\varepsilon^N$ which slips on Σ^0

Idea We use the Navier-slip condition

$$u_1 = \varepsilon\alpha\partial_2 u_1, \quad u_2 = 0 \text{ on } \Sigma^0 .$$

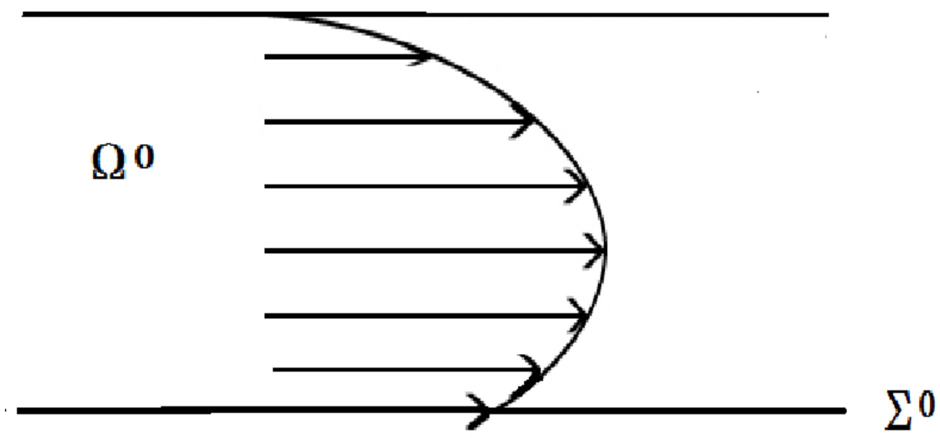


figure of $\vec{u}_\varepsilon^N$

The constant α depends only on the "shape" of the boundary.

Main Result

Theorem (Gérard-Varet・Masmoudi (2010))

The slipping flow $\vec{u}_{\varepsilon}^{N}$ approximates $\vec{u}^{\varepsilon}$ better than the no-slip flow $\vec{u}^{0}$.

Note

・Approximation rates are obtained in the order of ε

Key Ingredients

Difficulty

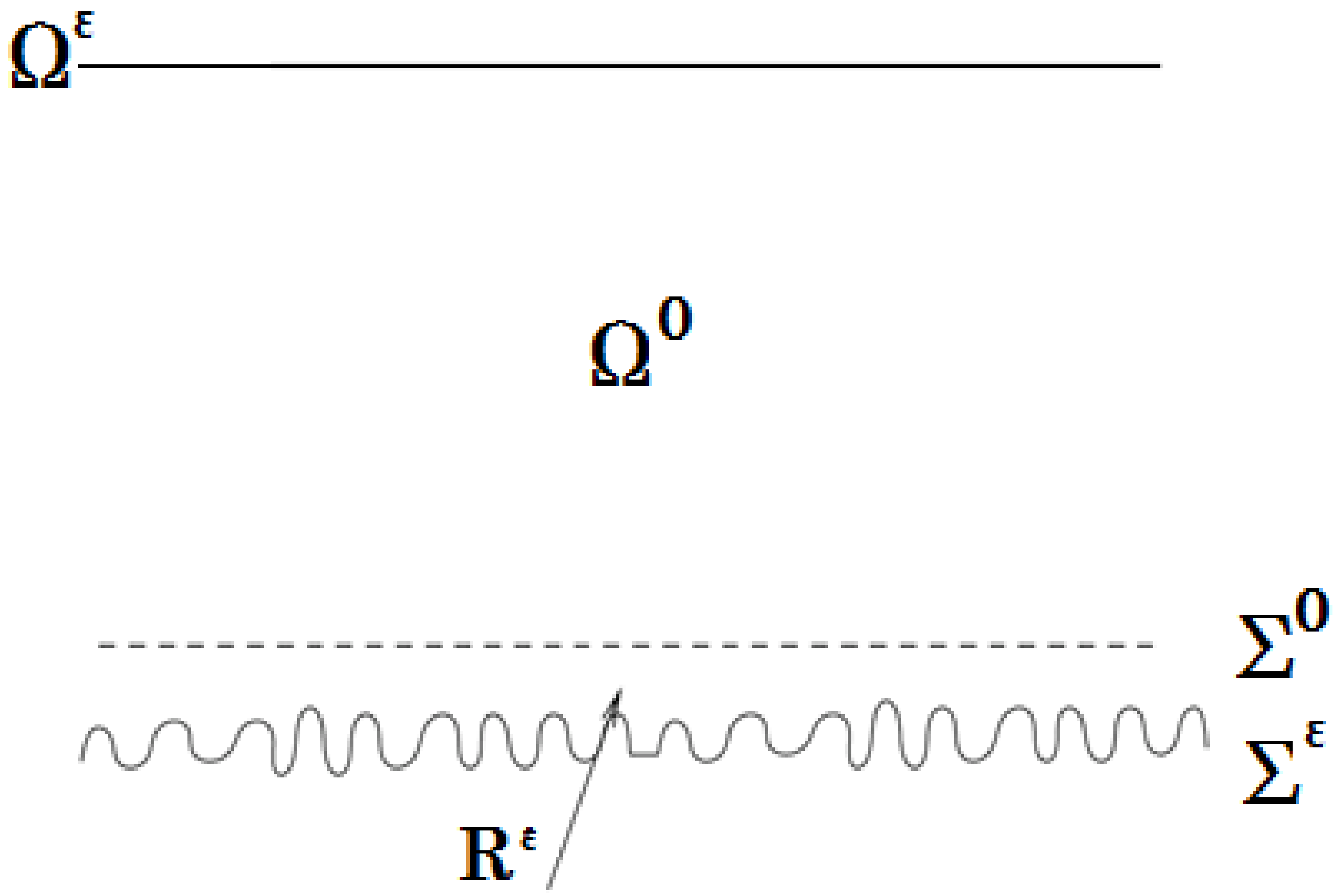
The derivation of the Navier-slip condition.

Particularly, the determination of the constant α.

Key Ingredients

Since the rough boundary oscillates, the difference of $\vec{u}^{\varepsilon}$ and $\vec{u}^{0}$ has an oscillating structure in R^{ε}.

A detailed analysis on this oscillating structure reveals the constant α.



Ω^{ε}
Ω^{0}
Σ^{0}
Σ^{ε}
R^{ε}

Conclusion

The boundary-roughness induces the slipping effect on the fluid dynamics, contrary to the no-slip condition on the boundary.

Further Study

・This argument describes the steady state

・Does the Navier-slip condition hold for time-changing flows?

⇒ Yes. But the initial state should be considered carefully.

Higaki, M. : Navier wall law for nonstationary viscous incompressible flows. JDE. **260** (10), 7358-7396 (2016).