

# **The 24th Northeastern Symposium on Mathematical Analysis**

Date: 20 - 21 February 2023

Venue: Kawai Hall, Graduate School of Science, Tohoku University

**Organizers:**

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**Website:**

[https://www.math.sci.hokudai.ac.jp/sympo/nema/24\\_en.html](https://www.math.sci.hokudai.ac.jp/sympo/nema/24_en.html)

This workshop is partially supported by

- Japan Society for the Promotion of Science, KAKENHI: Grant-in-Aid for Scientific Research (B) (No. 20H01812) “Evolution equations describing non-standard irreversible processes – Analysis on singularities emerging in the dynamics of solutions –”
- Institute for Mathematics in Advanced Interdisciplinary Study

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Venue : Kawai Hall, Graduate School of Science, Tohoku University  
(at Kita-Aobayama Campus)  
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<https://www.sci.tohoku.ac.jp/campusmap/kita-aobayama/>  
URL: [https://www.math.sci.hokudai.ac.jp/sympo/nema/24\\_en.html](https://www.math.sci.hokudai.ac.jp/sympo/nema/24_en.html)

## Program

### 20 February 2023

- 09:45 - 09:50      Opening
- 09:50 - 10:40      Takayoshi Ogawa (Tohoku University)  
Maximal  $L^1$  regularity for parabolic equations and a free boundary problem of the Navier–Stokes equations
- 10:50 - 11:40      Hiroaki Kikuchi (Tsuda University)  
Threshold solutions for the 3D focusing cubic-quintic nonlinear Schrödinger equation
- 11:40 - 13:10      Lunch (90min)
- 13:10 - 13:40      Tatsu-Hiko Miura (Hirosaki University)  
Error estimate for classical solutions to the heat equation in a moving thin domain and its limit equation
- 13:50 - 14:20      Kotaro Hisa (Tohoku University)  
Initial traces and solvability for a semilinear heat equation on a half space of  $\mathbb{R}^N$
- 14:30 - 15:00      Yusuke Ishigaki (Tokyo Institute of Technology)  
Asymptotic stability of stationary solutions to outflow problem for compressible viscoelastic system
- 15:15 - 15:35      Shuntaro Tsubouchi (The University of Tokyo, D2)  
Continuity of derivatives for certain very singular elliptic problems
- 15:35 - 15:55      Tatsuya Hosono (Tohoku University, D2)  
Global existence and  $(8\pi)^2$ -threshold of solutions to the 4D attraction-repulsion chemotaxis system
- 16:00 - 16:30      Poster Preview (Zoom)
- 16:30 - 18:30      Poster Session (Zoom)

## 21 February 2023

- 10:00 - 10:50 Hideo Kubo (Hokkaido University)  
Global existence and blow-up for nonlinear wave equations with inverse-square potential
- 11:00 - 11:30 Alessandro Palmieri (University of Bari)  
Blow-up results for a semilinear wave equation in the expanding de Sitter spacetime
- 11:40 - 12:10 Nobuhito Miyake (The University of Tokyo)  
Eventual global positivity of solutions to Cauchy problems of polyharmonic heat equations
- 12:10 - 13:40 Lunch (90min)
- 13:40 - 14:10 Takuya Sato (Tohoku University)  
The initial boundary value problem for the nonlinear Schrödinger equation with the nonlinear Neumann boundary condition in one space dimension
- 14:20 - 14:40 Motofumi Aoki (Tohoku University, D3)  
On the relationship between the Cauchy problem and the energy conservation law for the compressible Navier–Stokes equations
- 14:40 - 15:00 Shun Tsuhara (Tohoku University, D2)  
The boundary Strichartz estimates for the Schrödinger equation in the two-dimensional half plane and its application
- 15:00 - 15:20 Kiichi Tashiro (Tokyo Institute of Technology, M2)  
On the construction of canonical mean curvature flow by elliptic regularization
- 15:30 - 15:50 Poster Award Ceremony
- 16:00 - 16:50 Philippe Souplet (Université Sorbonne Paris Nord)  
Some recent Liouville type results and their applications (Zoom)
- 16:50 - 16:55 Closing

## Posters

- P1. Dáithí Ó hAodha (Tohoku University, D2)  
Large-Time Behaviour of Solutions to the Linearised Compressible Navier-Stokes Equations
- P2. Yudai Kanda (Tohoku University, M2)  
Gradient inequality for a Sobolev gradient flow and its application to Gross-Pitaevskii eigenvalue problem
- P3. Sho Katayama (The University of Tokyo, M2)  
Thresholds for the existence of solutions to supercritical elliptic problems
- P4. Shunsuke Kitamura (Tohoku University, D1)  
The lifespan estimates of classical solutions of one dimensional semilinear wave equations of derivative type with characteristic weights
- P5. Mizuki Kojima (Tokyo Institute of Technology, D1)  
On a time-fractional doubly critical equation, and its quantitative approach to the classical counterpart
- P6. Kenta Kumagai (Tokyo Institute of Technology, M2)  
Regularity of extremal solutions of semilinear elliptic equations with general nonlinearities
- P7. Fuya Hiroi (Tohoku University, M2)  
Curve diffusion flow for planar open curves on V-shaped polygonal lines
- P8. Kazuya Hirose (Hokkaido University, M2)  
A dynamical approach to lower gradient estimates for viscosity solutions of Hamilton-Jacobi equations
- P9. Shimpei Makida (Hokkaido University, D1)  
Stability of viscosity solutions on expanding networks
- P10. Hiroki Miyakawa (Tohoku University, M2)  
Maximal regularity for degenerate elliptic and parabolic equations of  $p$ -Laplacian type
- P11. Yoshihito Nakajima (Tohoku University, M2)  
Time-fractional evolution equations and applications to degenerate parabolic equations
- P12. Shozo Ogino (Tohoku University, M2)  
Singular limit problem of the initial value problem to the compressible Navier-Stokes equations in the critical Besov space
- P13. Yusuke Oka (The University of Tokyo, M2)  
Existence of solutions for time fractional semilinear parabolic equations in Besov–Morrey spaces
- P14. Akihito Ohgane (Hokkaido University, M2)  
Appropriate selection of sensory input produces good swing movement

- P15. Florian Salin (Tohoku University, D1)  
Implicit Scheme for Fractional Nonlinear Diffusion Equation Preserving Decay of Energy
- P16. Kotaro Sato (Tohoku University, D2)  
On some unidirectional evolution equation arising from fracture mechanics

# Abstracts

## **Maximal $L^1$ regularity for parabolic equations and a free boundary problem of the Navier-Stokes equations**

Takayoshi Ogawa (Tohoku University)

In this talk, we consider the free boundary problem of the incompressible Navier-Stokes equations near the half Euclidean space. We introduce maximal  $L^1$ -regularity estimate for the parabolic type equation first and apply it to the free boundary problem. The key idea for showing end-point maximal regularity is to obtain the almost orthogonal estimates between the Green's functions of the derivatives of the Stokes flow and the Littlewood-Paley dyadic decomposition of unity. The sharp boundary trace estimate as well as the inner and boundary bilinear estimates is used for the proof. This talk is based on the joint works with Prof. Senjo Shimizu (Kyoto University).

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## **Threshold solutions for the 3D focusing cubic-quintic nonlinear Schrödinger equation**

Hiroaki Kikuchi (Tsuda University)

In this talk, we consider the focusing cubic-quintic nonlinear Schrödinger equation in three space dimensions. Especially, we study the global dynamics of solutions whose energy and mass equal to those of the ground state in the sprits of Duyckaerts and Merle ('09). When we try to obtain the corresponding results of Duyckaerts and Merle, we meet several difficulties due to the cubic-quintic nonlinearity. We overcome them by using the one-pass theorem (no return theorem) developed by Nakanishi and Schlag ('12). This talk is based on a joint work with Masaru Hamano (Waseda University) and Minami Watanabe (Tsuda University).

**Error estimate for classical solutions to the heat equation  
in a moving thin domain and its limit equation**

Tatsu-Hiko Miura (Hirosaki University)

We consider the Neumann type problem of the heat equation in a moving thin domain around a given closed moving hypersurface. Our aim is to compare a solution to the thin domain problem with a solution to a limit equation on the moving hypersurface which appears in the thin-film limit of the heat equation. We show that the difference of classical solutions to the thin domain problem and to the limit equation is of order the thickness of the thin domain in the sup-norm. To prove such an error estimate, we derive an a priori estimate for a classical solution to the thin domain problem with a constant explicitly depending on the thickness of the thin domain. Based on that a priori estimate, we construct a suitable approximate solution to the thin domain problem from a classical solution to the limit equation by a formal asymptotic expansion of the thin domain problem.

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**Initial traces and solvability for a semilinear heat equation  
on a half space of  $\mathbb{R}^N$**

Kotaro Hisa (Tohoku University)

In this talk, we show the existence and the uniqueness of initial traces of nonnegative solutions to a semilinear heat equation on a half space of  $\mathbb{R}^N$  under the zero Dirichlet boundary condition. Furthermore, we obtain necessary conditions and sufficient conditions on the initial data for the solvability of the corresponding Cauchy–Dirichlet problem. Our necessary conditions and sufficient conditions are sharp and enable us to find optimal singularities of initial data for the solvability of the Cauchy–Dirichlet problem. This talk is based on a joint work with Prof. K. Ishige (The University of Tokyo) and Prof. J. Takahashi (Tokyo Institute of Technology).

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**Asymptotic stability of stationary solutions to outflow problem  
for compressible viscoelastic system**

Yusuke Ishigaki (Tokyo Institute of Technology)

We investigate the existence and stability of stationary solutions to the outflow problem for compressible viscoelastic system in the one-dimensional half space. We classify the existence of stationary solutions by determining suitable conditions for several parameters, such as the Mach number and propagation speed of elastic wave. We next establish its stability result under the small initial perturbation. This talk is based on a joint work with Yoshihiro Ueda (Kobe University).



## Continuity of derivatives for certain very singular elliptic problems

Shuntaro Tsubouchi (The University of Tokyo)

In this talk, we consider a one-Laplace equation or system perturbed by  $p$ -Laplacian. These elliptic problems become no longer uniformly elliptic near a facet, the place where a gradient degenerates. Recently, the speaker found it possible to prove continuous differentiability of weak solutions even across facets. The aim of this talk is to explain briefly how gradient continuity is shown.

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## Global existence and $(8\pi)^2$ -threshold of solutions to the 4D attraction-repulsion chemotaxis system

Tatsuya Hosono (Tohoku University)

We consider the Cauchy problem of an attraction-repulsion chemotaxis system, which was proposed as a chemotaxis model in response to a combination of chemoattractant and chemorepellent secreted by the cells themselves. One of main topics in the study of such a system is the presence of the  $L^1$  threshold. Indeed, critical mass phenomena called “ $8\pi$ -problem” are well-known in the 2-dimensional case. In this talk we establish that the solution exists globally in time under the 4-dimensional setting, provided that the initial mass is less than the 4-dimensional  $L^1$  threshold value  $(8\pi)^2$ . To this end, we introduce the Brezis–Merle type inequality for the 4th-order elliptic equation and rearrangement arguments. This talk is based on a joint work with Professor Takayoshi Ogawa (Tohoku University).

## **Global existence and blow-up for nonlinear wave equations with inverse-square potential**

Hideo Kubo (Hokkaido University)

In this talk I'd like to present blow-up and global existence results for the Cauchy problem to the nonlinear wave equation with the inverse-square potential. Because of the strong singularity of the inverse-square potential, one may expect the feature of the problem becomes quite different from that of the corresponding problem without potential. However, as long as small amplitude solutions are concerned, we will see that the critical exponent of the nonlinearity which separates the blow-up and global existence is unchanged, though we would meet technical difficulties. The key point of our analysis is the study of Rellich type inequality for the Laplacian with the inverse-square potential. In fact, it provides us nice alternative to Klainerman's inequality which is the one of the most important tool for the study of the nonlinear wave equation. This talk is based on the joint works with Prof. V. Georgiev, Prof. M. Sobajima, and Prof. W. Dai.

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## **Blow-up results for a semilinear wave equation in the expanding de Sitter spacetime**

Alessandro Palmieri (University of Bari)

In this talk, we consider a semilinear wave equation in the expanding de Sitter spacetime. We will focus on blow-up results and upper-bound estimates for the lifespan of local solutions to the corresponding Cauchy problem. The technique in the proof of these results is based on the classical approach for the wave equation in the flat case, which consists in studying the growth of the time-dependent function given by the spatial average of a local solution. A special emphasis will be put on a threshold case for the parameters appearing in the nonlinear term: we will combine an integral representation formula by Yagdjian-Galstian with a technique borrowed from the critical case for the classical wave equation. Based on a joint work with Professor Hiroyuki Takamura (Tohoku University).

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## **Eventual global positivity of solutions to Cauchy problems of polyharmonic heat equations**

Nobuhito Miyake (The University of Tokyo)

In this talk, we consider Cauchy problems of polyharmonic heat equations. Our aim of this talk is to show the existence of the threshold for the decay rate of initial data which determines whether the corresponding solution to the Cauchy problem of the linear polyharmonic heat equation is eventually globally positive or not. As the application of this result, we construct eventually globally positive solutions to a semilinear polyharmonic heat equation.

**The initial boundary value problem for the nonlinear Schrödinger equation with the nonlinear Neumann boundary condition in one space dimension**

Takuya Sato (Tohoku University)

We consider the initial boundary value problem of the nonlinear Schrödinger equation on the half line with a nonlinear Neumann boundary condition in one space dimension. After establishing the boundary Strichartz estimate in  $L^2$ , we consider the time local well-posedness of the problem in the lower regularity spaces as  $L^2(\mathbb{R}_+)$ .

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**On the relationship between the Cauchy problem and the energy conservation law for the compressible Navier–Stokes equations**

Motofumi Aoki (Tohoku University)

In this talk, we consider the energy conservation law of the full system of compressible Navier–Stokes equations. The existence of weak solutions is known. We study sufficient conditions that the weak solutions satisfy energy conservation law. We also study the ill-posedness of the Cauchy problem for the equations of the ideal gas in the critical Besov spaces and compare the ill-posedness condition with the sufficient conditions for energy conservation law. This talk is based on a joint work with Professor Tsukasa Iwabuchi (Tohoku University).

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**The boundary Strichartz estimates for the Schrödinger equation in the two-dimensional half plane and its application**

Shun Tsuhara (Tohoku University)

We consider the initial boundary value problem for the nonlinear Schrödinger equation in the half plane with the nonlinear Neumann boundary condition. In the previous works, Batal–Özsari and Hayashi–Ogawa–Sato considered such a problem for the one-dimensional case. In this talk, we show the boundary Strichartz estimates for the two-dimensional case with the Neumann boundary term and local well-posedness for the nonlinear Schrödinger equation in  $L^2(\mathbb{R}_+^2)$ . This talk is based on a joint work with Prof. Takayoshi Ogawa and Prof. Takuya Sato (Tohoku University).

## On the construction of canonical mean curvature flow by elliptic regularization

Kiichi Tashiro (Tokyo Institute of Technology)

In this talk, we discuss the generalized mean curvature flow constructed by elliptic-in-time regularization. Generally, the generalized flows have a non-uniqueness issue, such as suddenly disappearing. To this problem, we show the validity of an explicit identity about the area change of the evolving grain. Such additional property resolves the non-uniqueness of the generalized flows under suitable assumptions on the initial datum.

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## Some recent Liouville type results and their applications

Philippe Souplet (LAGA, Université Sorbonne Paris Nord)

We present some recent Liouville type theorems for nonlinear elliptic equations, especially in the case of a half-space. The problems under consideration include the Lane-Emden equation and the diffusive Hamilton-Jacobi (DHJ) equation. In the latter case, we will discuss applications of the elliptic Liouville type theorem to the asymptotics of gradient blow-up and to the loss of boundary conditions for viscosity solutions of the parabolic DHJ equation, which arises in stochastic optimal control.

Joint works with L. Dupaigne, R. Filippucci, P. Pucci and B. Sirakov.