# The 23rd Northeastern Symposium on Mathematical Analysis

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## Abstracts

**Kentaro Fujie (Tohoku University)**

**Boundedness of solutions to a fully parabolic chemotaxis system with local sensing in higher dimensions**

We consider some parabolic-parabolic chemotaxis system with local sensing in higher dimensions. We will establish that classical solutions of the system exist globally in time and remain uniformly bounded in time if the decay rate of motility functions is less than some constant, independently of the magnitude of mass. We will introduce some auxiliary function and prove that this function satisfies some evolution equation, which helps us to derive enough information about the original system. The cornerstone of the analysis is the refined comparison estimate for solutions. This talk is based on joint works with Takasi Senba (Fukuoka University).

**Nakao Hayashi (Tohoku University)**

**Initial boundary value problem for nonlinear Schrödinger equations**

We consider the initial-boundary value problem for nonlinear Schrödinger equations on the half-line with the nonlinear Neumann boundary condition. We show a existence of $L^2$ solutions to the integral equation associated with the original problem. This is a joint work with T.Ogawa and T. Sato (Tohoku University).

**Yucong Huang (Oxford University/Tokyo Institute of Technology)**

**On Spherically Symmetric Weak Solution to the Cauchy Problem for Compressible Full Navier-Stokes Equations**

In this talk, I will discuss the existence of a spherically symmetric weak solution to the Cauchy problem for compressible full Navier-Stokes equations. Although the symmetry assumption simplifies many aspects of the problem, it also introduces coordinate singularity at the origin, indicated by the terms 1/r in the equations. To circumvent this issue, a set of initial-boundary value problems called exterior sphere problems is introduced, which corresponds to the physical model where an insulating ball of finite radius, $a > 0$, is centred at the origin. Many progresses have been made on the exterior sphere problem by various authors (See [1,2,5]). However, significant difficulties arise when obtaining solution to the

original problem via the limit process of $a > 0$ goes to 0. One of the first results on this was made by Hoff in [3], where he constructed such solution for the isentropic case. Later, Hoff and Jenssen in [4] extended this framework to the heat-conducting compressible flow, however it is restricted to the initial-boundary value problem posed on a finite ball, and some major issues are still present regarding the dependence on the inner boundary radius $a > 0$. The main result presented in this talk will not only extend their result to the Cauchy problem with a non-vacuum state at the spatial infinity, but I will also provide a resolution to the issue of the dependence on $a > 0$. The main strategy is to regard the Cauchy problem as the limit of a series of initial-boundary value problems that are formulated in finite annular regions under the Lagrangian coordinates, and the key argument is to establish a set of a-priori estimates that are uniform with respect to both the inner and outer radii of the annuli considered.

[1] H. Fujita-Yashima and R. Benabidallah, Unicité de la solution de l'équation monodimensionnelle ou á symétrie sphérique d'un gaz visqueux er calorifére, Rendiconti del Circolo Mat. di Palermo 42 (1993), 195-218.

[2] H. Fujita-Yashima and R. Benabidallah, Equation á symétrie sphérique d'un gaz visqueux et calorifére avec la surface libre, Ann. di Mat. Pura ed Appl. 168 (1995), 75-117.

[3] D. Hoff, Spherically symmetric solutions of the Navier-Stokes equations for compressible, isothermal flow with large, discontinuous initial data, Indiana Univ. Math. J. 41 (1992), 1225-1302.

[4] D. Hoff and H. K. Jenssen, Symmetric nonbarotropic flows with large data and forces, Arch. Ration. Mech. Anal. 173 (2004), 297-343.

[5] S. Jiang, Global spherically symmetric solutions to the equations of a viscous polytropic ideal gas in an exterior domain, Comm. Math. Phys. 178 (1996), 339-374.

**Hiroshi Ishii (Hokkaido University)**

**Asymptotic profiles of zero points of solutions to nonlocal diffusion equations**

In this talk, we consider the Cauchy problem for nonlocal diffusion equations with rapidly decaying smooth integral kernels. The asymptotic behavior of zero points of the solution is investigated, and we report the results of deriving its asymptotic profiles up to a constant term.

**Shuichi Jimbo (Hokkaido University)**

**A Semilinear parabolic equation in a simple unbounded metric graph**

I consider the dynamics of a semilinear parabolic in a simple metric graph which is obtained by connecting several half lines (and possibly finite line segments). I deal with the initial value problems and their asymptotic behaviors in time variable and study the existence of stationary solutions. Particularly I consider the waves which move on branches and study the structure of the flows around stationary solutions.

**Takashi Kagaya (Muroran Institute of Technology )**

**Singular Neumann boundary problems for a class of fully nonlinear parabolic equations**

We discuss singular Neumann boundary problem for a class of nonlinear parabolic equations. Our boundary problem describes motion of a planar curve sliding along the boundary with a zero contact angle, which can be viewed as a limiting model for the capillary phenomenon. We study the existence and non-existence of bounded solutions by using the viscosity solution theory.

**Shota Sakamoto (Tokyo Institute of Technology)**

**A Cauchy problem of the non-cutoff Boltzmann equation in a Fourier-Lebesgue space in $\mathbb{R}^3$**

A Cauchy problem of the Boltzmann equation without angular cutoff is studied in $\mathbb{R}^3$. For the three dimensional torus $\mathbb{T}^3$ case, a unique solution is obtained in the Wiener space, which we denote $L^1_k$. The obvious substitution of this space for $\mathbb{R}^3$ is the space of functions whose Fourier transform is $L^1$. Due to the lack of sufficient dissipation, this norm does not close the whole energy estimate. To overcome this, we employ $L^1$-$L^\infty$ interplay in the Fourier space, and we can obtain a unique global solution to this case.

**Megumi Sano (Hiroshima University/ Tohoku University)**

**Improvements and generalizations of two Hardy type inequalities**

We give improvements and generalizations of both the classical Hardy inequality and the geometric Hardy inequality based on the divergence theorem. Our improved Hardy type inequality derives both two Hardy type inequalities with best constants.

**Daniel Spector (National Taiwan Normal University)**

**Sobolev Inequalities in $L^1$**

The study of Sobolev Inequalities in $L^1$ has now been under way for more than half a century. In this talk, I will give a survey of the results in this area, including the principle contributors, what they have helped us to understand so far, and what the frontier of research is at present. Open problems will be mentioned.

**Takeshi Suguro (Tohoku University)**

**Singular limit problem for the Keller-Segel system in uniformly local spaces**

We consider the Cauchy problem of the Keller–Segel system on the uniformly local Lebesgue space. The system has a nonlocal property in its solution and it is a subtle question if it is well-posed in the local function class. We first establish the local well-posedness of the system in the scaling critical uniformly local Lebesgue space and consider the zero relaxation time limit for the system. Such a singular limit reduces the system into the drift-diffusion equation of the parabolic-elliptic type. To show the singular limit, we invoke maximal regularity for the Cauchy problem of the heat equation on the non-reflexive real interpolation space involving the local Morrey and the local Kozono–Yamazaki spaces. This talk is based on a joint work with Professor Takayoshi Ogawa (Tohoku University).

**Satoshi Tanaka (Tohoku University)**

**On the uniqueness of positive radial solutions to superlinear elliptic equations in annuli**

In this talk, a class of superlinear elliptic equations in annuli is considered. A generalized comparison identity for a two-point boundary value problem is introduced. By applying it, the uniqueness results of positive radial solutions are established. Our results improve some classical results. This is joint work with Noaki Shioji (Yokohama National University), Kohtaro Watanabe (National Defense Academy), Carmen Cortazar, Marta Garcia-Huidobro, and Pilar Herreros (Pontificia Universidad Catolica de Chile).

**Hiroshi Wakui (Tokyo University of Science)**

**Stability of constant steady states of a parabolic-elliptic Keller-Segel system**

We consider stability of constant steady states of a parabolic-elliptic Keller-Segel system on the n dimensional Euclidian spaces. Due to the structure of our problem, we have infinitely many constant steady states of our problem. We will show that stability of each constant solutions depends on the range of its value.

**Shuangquan Xie (Tohoku University)**

**Oscillatory localized patterns in reaction-diffusion systems**

In this talk, we introduce a three-component reaction-diffusion model, whose key feature is that it has a solution consisting of N spikes that undergoes Hopf bifurcations with respect to N distinct modes nearly simultaneously. This results in complex oscillatory dynamics of the spikes, not seen in typical two-component models. We show that for small-amplitude oscillations in the spike position induced by multiple Hopf modes, the long-time behavior can be one of two types of stable oscillations. Both of those two types of oscillations are stable, coexist for the same parameter values, and the fate of motion depends solely on the initial conditions.

**Keisuke Yoshida (Hokkaido University)**

**Simplicity of C*-algebras associated to self-similar groups**

Self-similar groups are some kinds of groups consisting of homeomorphisms on the Cantor space (we consider symbolic dynamics). By definition, self-similar groups are equipped with relations between elements in groups and one-sided shift maps. Nekrashevych constructed C$^*$-algebras respecting the relations. I am interested to obtain invariants of self-similar groups through ones of the C$^*$-algebras. To discuss the invariants of C$^*$-algebras, we sometimes require the simplicity (i.e no non-trivial ideal) of C$^*$-algebras. I will talk about a necessary and sufficient condition for the simplicity of C$^*$-algebras.

**Kensuke Yoshizawa (Tohoku University)**

**An obstacle problem for the elastic energy among graph curves pinned at endpoints**

This talk is devoted to an obstacle problem for the $p$-elastic energy among graph curves with fixed ends. There are two causes of the loss of regularity of solutions to the obstacle problem: one is 'degeneracy' of the Euler–Lagrange equation; the other is the presence of the 'obstacle'. In this talk we address the question which is the main cause of the loss of regularity. This talk is based on a joint work with Prof. Anna Dall'Acqua (Universität Ulm), Dr. Marius Müller (Albert-Ludwigs-Unversität Freiburg), and Prof. Shinya Okabe (Tohoku University).