# Stable phase interfaces in the van der Waals - Cahn – Hilliard theory

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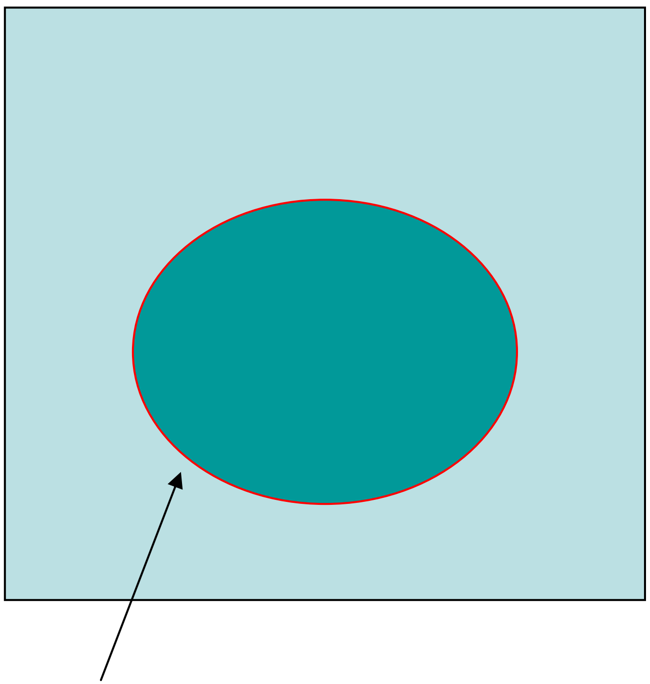
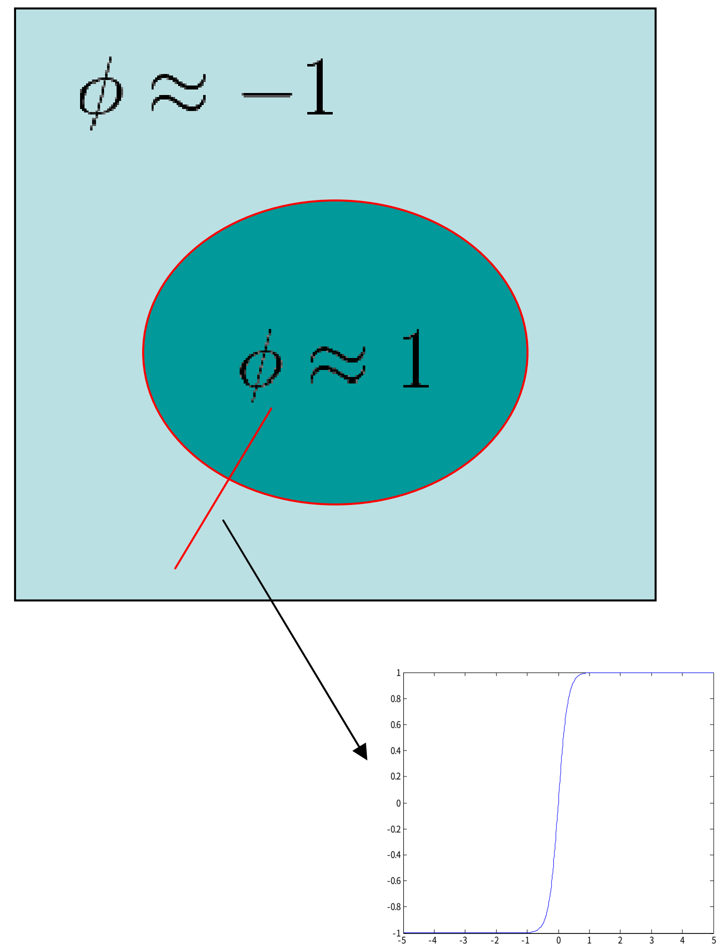
- Diffused interface model
  (or phase field model)



Sharp Interface $\Gamma$



Interface area of $\Gamma \subset \mathbb{R}^n$ $\longrightarrow$

n-1dimensional Hausdorff measure $\mathcal{H}^{n-1}(\Gamma)$

Diffused interface model

$$\mathcal{H}^{n-1}(\Gamma) \iff E_\varepsilon(\phi) = \int_\Omega \varepsilon \frac{|\nabla \phi|^2}{2} + \frac{W(\phi)}{\varepsilon} dx$$

- $\varepsilon > 0$ is a small parameter of O(interface thickness).
- $W(\phi) = (1 - \phi^2)^2/4$ : double-well potential.

- Known results (relevant to our results)
  - Gamma convergence for energy minimizers

    Modica-Mortola ’77, Modica ’87, Sternberg ’88…
  - Critical points (not necessarily minimizers)

    Hutchinson-T. ’00, T. ’05…
  - Gradient flow (Allen-Cahn equation)

    Bronsard-Kohn ’91, X.-F. Chen ’93, Ilmanen ’93…

Whole lots of related results!

Theorem (T.- Wickramasekera, ’10)

Suppose $\{\phi_i\}_{i\geq 1}$ and $\varepsilon_i \to 0$ satisfy

- $E_{\varepsilon_i}(\phi_i) \leq \exists C$ and $|\phi_i| \leq \exists C$
- $\phi_i$ are critical points: i.e. $-\varepsilon_i \Delta\phi_i + \frac{W'(\phi_i)}{\varepsilon_i} = 0$
- $\phi_i$ are stable: i.e. the second var. ≥ 0

Conclusion:

For any 0≤s<1 and for some subsequence,

$\{|\phi_{i_j}| < s\} \to \Gamma$ in Hausdorff distance and $\Gamma$ is an embedded smooth stable minimal hypersurface if 1<n<8.