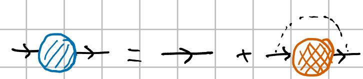
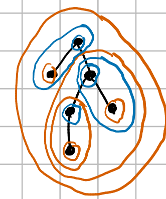
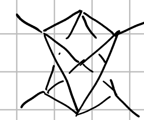
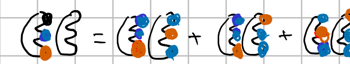
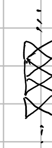
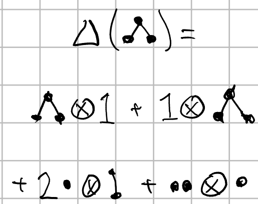




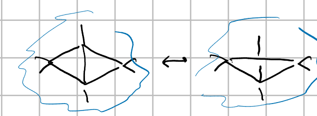




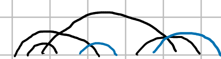




$\phi B_+ = \Lambda \phi$

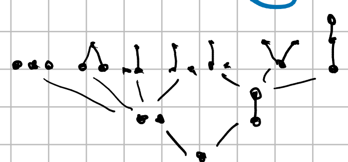


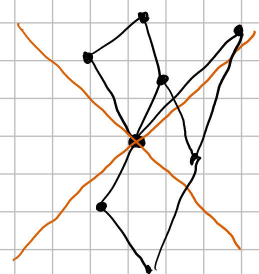
Not these stories

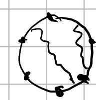


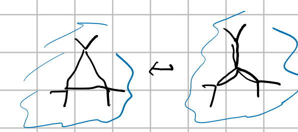
This story

$M_{\times} = M_{=} + M_{||} + M_{\times}$









Combinatorics and Causal Set Theory

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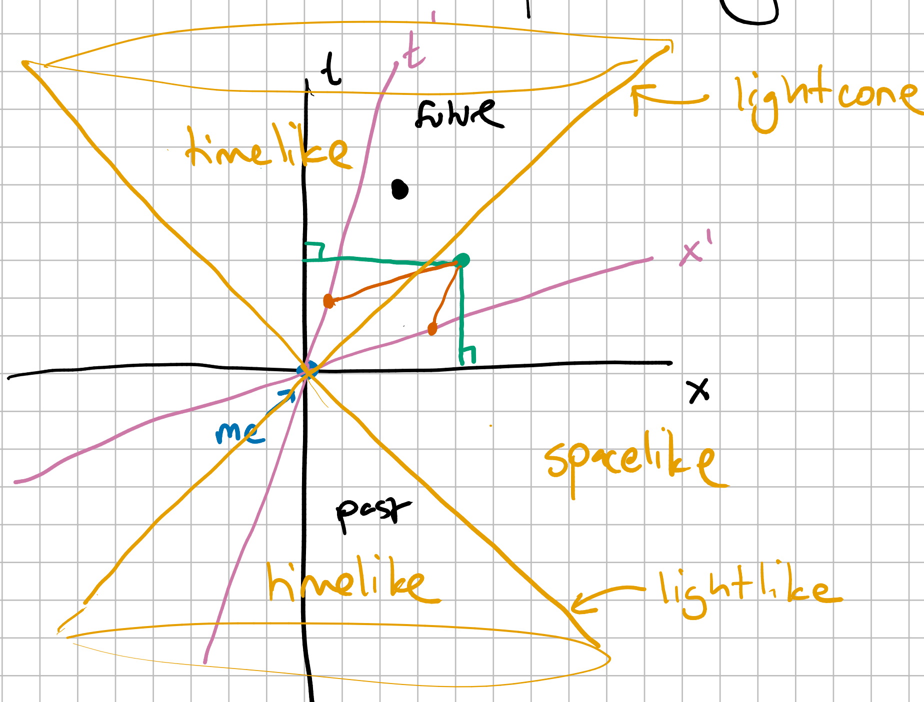
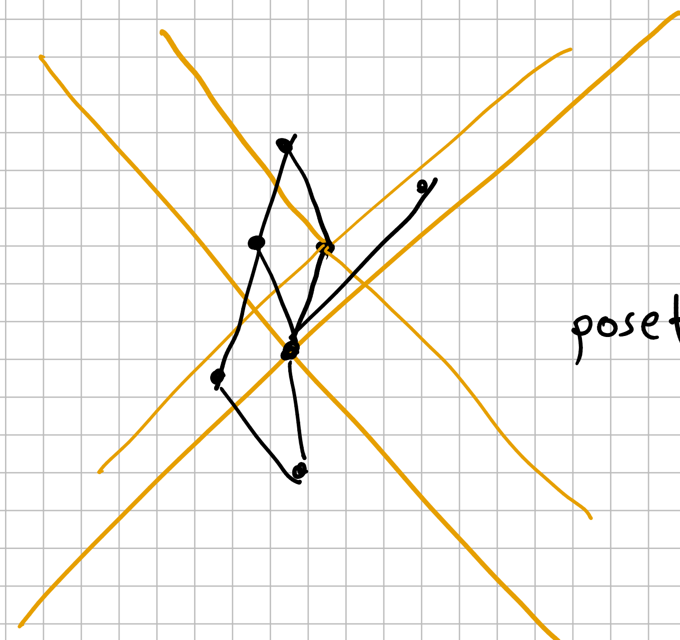
① Causal Set theory

Myrheim '78
Bombelli, Lee, Meyer, Sorkin '87

One day I was sitting in a quantum gravity talk that ended up being a poset talk.





Poset language

Poset axioms $(P, \leq)$

with $a \leq a$

$a \leq b\ ,\ b \leq a \Rightarrow a = b$

$a \leq b\ ,\ b \leq c \Rightarrow a \leq c$

Interval $[a, b] = \{c : a \leq c \leq b\}$

P is locally finite if

all intervals are finite

Downset generated by $a \in P$

$= \{b : b \leq a\}$

Upset generated by $a \in P$

$= \{b : b \geq a\}$

Causal set language

P is a causal set

Past of a

Future of a

Number of points in an interval   Volume of a spacetime interval

Can this be enough?

Even if not surely we have at least this

Sorkin's '91
Lorentzian, discrete, path integral
fork in the road

For suitably nice continuous spacetimes it is
Hawking, King, McCarthy, Malament
'76 '77

If a causal set is suitably manifold-like, then up to the discreteness scale does it determine its manifold up to isometry

What can we concretely do?

② Poset invariants with physical analogues

height = length of longest chain becomes geodesic distance

In finite case can enumerate pairs $(|\text{past}(x)|, |\text{future}(x)|)$
can this see manifoldness at least locally

Sorkin, in progress George, —

What can a generating series of interval size see?

dimension if came from a Minkowski space

how much more? Surya, in progress George, —

Other dimension estimators beginning with

Myrheim '78 $\dfrac{2(\#\text{relations})}{|P|(|P|-1)}$ depends only on dimension

with same caveats

But what does came from a Minkowski space mean?

(3) Sprinklings

Take a Lorentzian manifold

Choose points on it by a Poisson distibution

Use the induced causal order

Get a poset with

$$P\begin{pmatrix} m \text{ poset elements} \\ \text{in volume } V \end{pmatrix} = \frac{(\rho V)^m e^{-\rho V}}{m!}$$

density parameter

Mathematics of such random posets Brightwell, Luczak and more

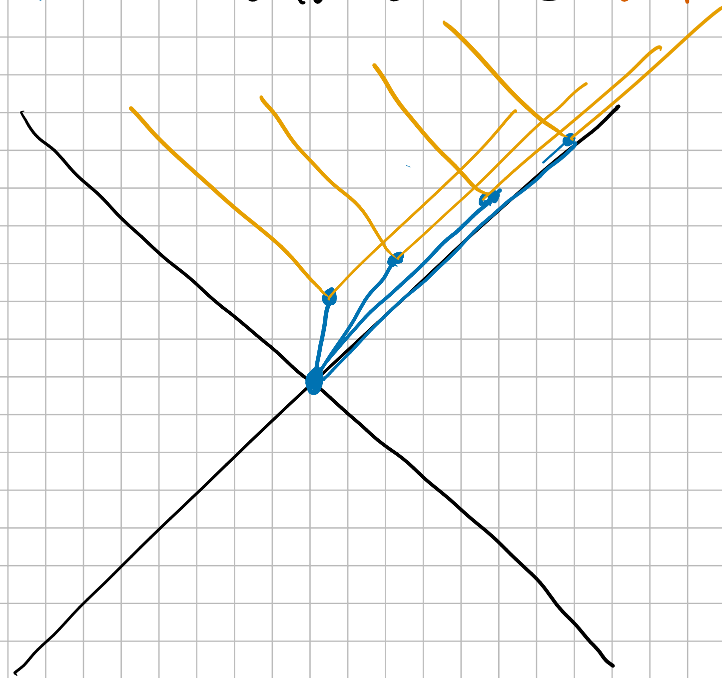
Many connections eg height of random 2d causal set is longest increasing subsequence in a random permutation

Eg would like causal set analogue of the d'Alembertian

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$$

Differential operator should become difference operator

but



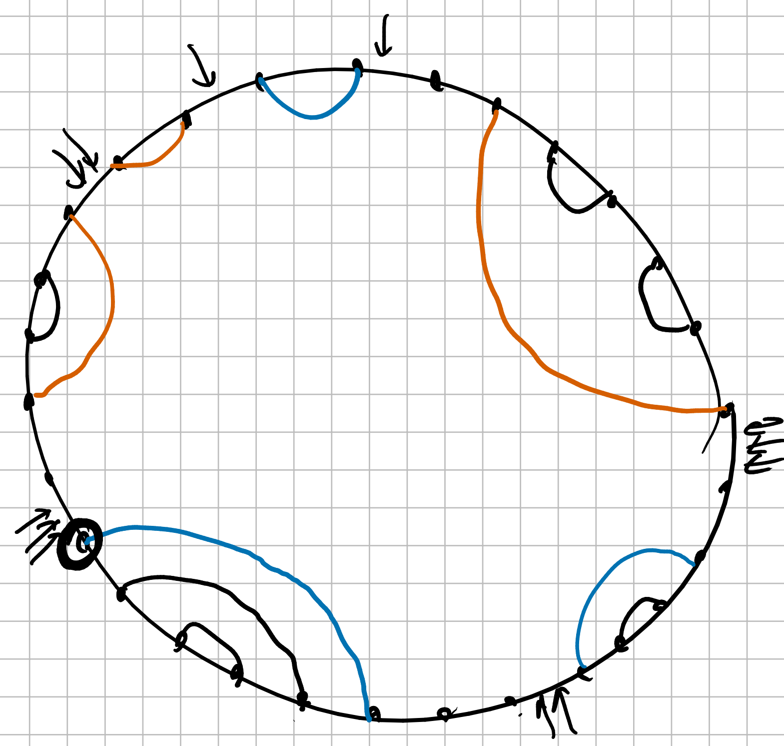
Use an alternating sum

Sorkin '09 in 2d, Benincasa, Dowker '10 in 4d

Dowker, Glaser '13, '14 in general

Get	C_1	C_2	C_3	C_4
$d=1$	1	$-\frac{1}{2}$		
$d=2$	1	-2	1	
$d=3$	1	$-\frac{27}{8}$	$\frac{9}{4}$	
$d=4$	1	-9	16	-8

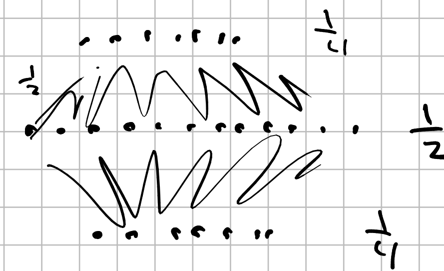
Combinatorial interpretation —



But we don't want to start with a manifold

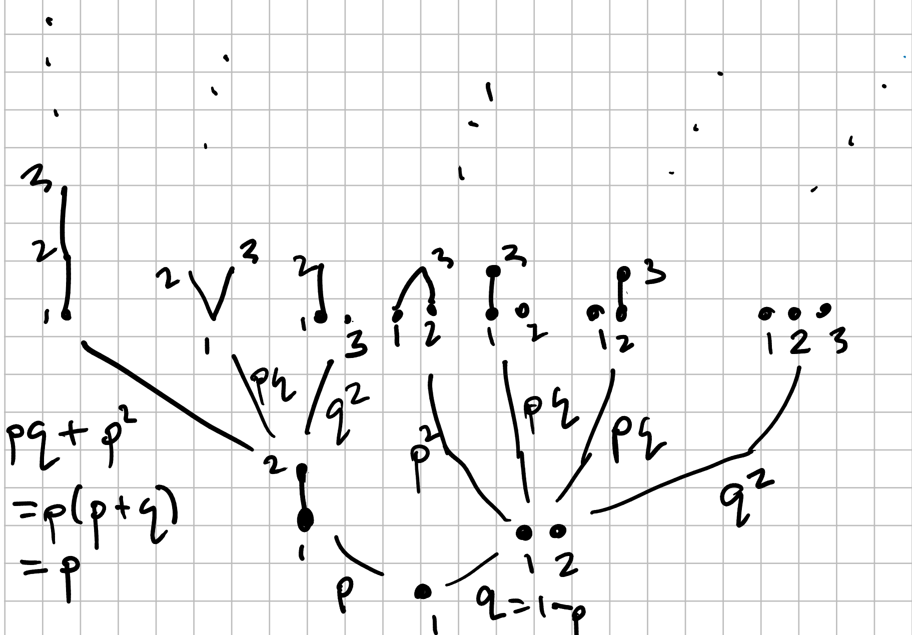
④ Growth models

Not uniformly at random



Kleitman, Rothschild '75

So grow them



$pq + p^2$
$= p(p+q)$
$= p$

Transitive percolation

=

random graph order

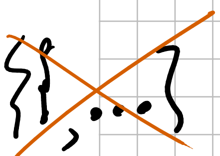
Alon, Bollobás, Brightwell, Janson and more including Gao

Weighting differently get classical sequential growth
Rideout, Sorkin

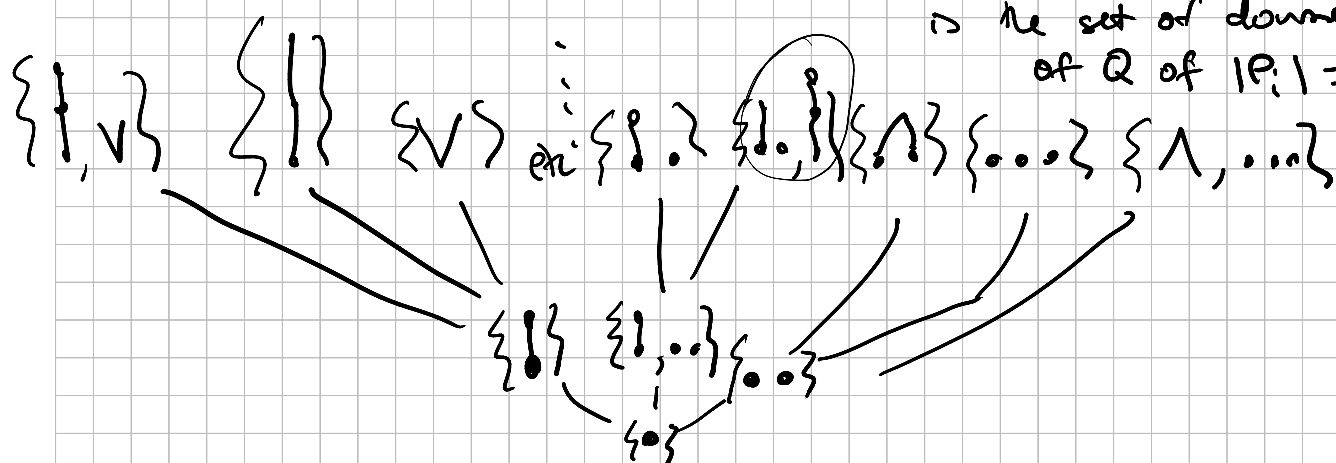
the models satisfying general covariance, Bell causality and gregariousness.

If we want covariance throughout = unlabelled

1. get covtree Dowker, Imambaccus, Owens, Sorkin, Zalel '19



$\{P_1, \dots P_k\}$ in covtree if $\exists Q$ such that $\{P_1, \dots P_k\}$ is the set of downsets of Q of $|P_i| = |P_j|$



$\{Q_1, \dots, Q_\ell\}$ $|Q_i| = |Q_j|$

$\{P_1, \dots, P_k\}$ $|P_i| = |P_j|$

if $\{P_1, \dots, P_k\}$ are the downsets of size $|P_i| = |Q_i| - 1$ of $\{Q_1, \dots, Q_\ell\}$

⑤ So what?

as a combinatoricist

- physics asks rich, unexpected questions in pure combinatorics
- not overworked for mathematicians
- give back to physics

as a physicist

- start with simple physically motivated axioms
- new progress from new interest by mathematicians.