On the turning point problem for Painlevé equations with a large parameter

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In this talk we will discuss the turning point problem for Painlevé equations (P_J) $(J = \mathrm{I}, \ldots, \mathrm{VI})$ with a large parameter.

As is well known, a second order linear ordinary differential equation is transformed to the Airy equation near a simple turning point. In a similar manner, it is transformed to the Weber equation and to the Whittaker equation near a double turning point and near a simple pole, respectively. Our purpose is to generalize these results to Painlevé equations.

In [1] we proved that any Painlevé equation (P_J) is transformed to the first Painlevé equation (P_{I}) near a simple turning point. To be more specific, every formal instanton-type solution (or transseries solution) of (P_J) is transformed to that of the first Painlevé equation near its simple turning point. This result can be considered as a counterpart of the above result for a second order linear differential equation near a simple turning point. Then, what are the counterparts of the above results near a double turning point and a simple pole? Our answer to this question is given by the following

Theorem 1. *Near a double turning point every formal instanton-type solution of* (P_J) *is transformed to that of the* (*degenerate*) *second Painlevé equation.*

Theorem 2. *Near a simple pole every formal instanton-type solution of* (P_J) *is transformed to that of the third Painlevé equation of type* $(D8)$*, that is, the most degenerate third Painlevé equation.*

In the talk we will explain the precise meaning of these theorems and also some recent results related to them.

References

[1] T. Kawai and Y. Takei, WKB analysis of Painlevé transcendents with a large parameter. III, *Adv. Math.*, **134**(1998), 178–218.

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[3] Y. Takei, On the role of the degenerate third Painlevé equation of type $(D8)$ in the exact WKB analysis, *RIMS Kôkyûroku Bessatsu*, **B37**(2013), 211–222.