Encoding of streamline topologies for incompressible vortex flows in 2D multiply connected domains

Takashi Sakajo
Department of mathematics, Kyoto University, Kyoto, Japan
Japan Science and Technology CREST, Tokyo, Japan
E-mail: sakajo@math.kyoto-u.ac.jp

Let us consider the flows generated by many vortex structures in the presence of the uniform flow in two-dimensional multiply connected exterior domains. They are regarded as mathematical models for biofluids such as insect flights and vertical descend of rotating plant seeds and for environmental flows in rivers and coastal flows. For the sake of theoretical simplicity, we suppose that the fluid is incompressible and inviscid (or slightly viscid). Then the instantaneous flow field $\boldsymbol{u}(t,x,y) = (u(t,x,y), v(t,x,y))$ in the two-dimensional (x,y)-plane is constructed from the stream function $\psi(t,x,y)$ by $u = \partial_y\psi$ and $v = -\partial_x\psi$, whose corresponding streamlines correspond to the contour lines of the stream function.

Here, we are concerned with the global topological structure of the streamline patterns generated by the vortex structures in the uniform flow. Classification of streamline patterns for the unbounded plane has been investigated by Aref and Brøns[1] and that for the sphere was done by Kidambi and Newton[2], in which no boundary is contained in the flow domains. The present study is not only an extension to the case of planar flow domains with many boundaries, but it also adds the following new aspects that have not been considered so far. First, the uniform flow is taken into considerations. This is an essential element in the study of biofluids and environmental flows, and it adds a new streamline structure to the flow profile topologically. Second we focus on *the structurally stable vector fields*, which are of significance since the structurally stable flows are more probable to be observed in many real flow phenomena. Third, the existence of separation points at the boundaries appends another structurally stable streamline structure. Thus the topological classification of the streamline patterns in multiply connected domains provide us with non-trivial new results mathematically that are applicable to many fluid phenomena.

In the present talk, let us first introduce an inductive procedure to construct structurally stable streamline patterns generated by finitely many vortex structures in the presence of the uniform flow[3]. That is to say, starting from some basic structurally stable streamline patterns in the disk of low genus, we repeat some fundamental operations that append a streamline pattern with increasing one genus to them. Owing to the procedure, one can regard a sequence of the operations as *a representing word* of each structurally stable streamline pattern. We then give an encoding algorithm to assign an unique canonical word expression for a given structurally stable vector field, which allows us to determine all possible structurally stable streamline patterns in a combinatorial manner. Finally the applications of the present encoding theory to some fluid problems are provided.

References

[1] H. Aref and M. Brøns 1998 On stagnation points and streamline topology in vortex flows. *J. Fluid Mech.* **370**, 1–27.

[2] R. Kidambi and P. K. Newton 2000 Streamline topologies for integrable vortex motion on a sphere. *Physica D* **140**, 95–125.

[3] T. Yokoyama and T. Sakajo 2013 Word representation of streamline topologies for structurally stable vortex flows in multiply connected domains. *Proc. Roy. Soc. A* **469** doi: 10.1098/rspa.2012.0558