Existence and stability of time-periodic solution of the compressible Navier-Stokes equation

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We consider a time periodic problem of the following compressible Navier-Stokes equation in $\mathbb{R}^n$ $(n \geq 3)$:

$$\begin{cases} \partial_t \rho + \mathrm{div}\,(\rho v) = 0, \\ \rho(\partial_t v + v \cdot \nabla v) - \mu \triangle v - (\mu + \mu')\nabla \mathrm{div}\, v + \nabla P(\rho) = \rho g. \end{cases} \tag{1.1}$$

Here, $\rho = \rho(x,t)$ and $v = (v^1(x,t), \cdots, v^n(x,t))$ denote the unknown density and the unknown velocity field, respectively, at time $t \geq 0$ and position $x \in \mathbb{R}^n$; $P = P(\rho)$ is the pressure that is assumed to be a smooth function of ρ satisfying $P'(\rho_*) > 0$ for a given constant $\rho_* > 0$; μ, μ' are the viscosity coefficients that are assumed to be constants and satisfy $\mu > 0$ and $\frac{2}{n}\mu + \mu' \geq 0$; and $g = g(x,t)$ is a given external force periodic in t with period $T > 0$.

The purpose of this talk is to investigate the existence and stability of a time-periodic solution of system (1.1).

Ma, Ukai, and Yang (2010) [1] showed that if $n \geq 5$, there exists a time-periodic solution $(\rho_{per}(t), v_{per}(t))$ around $(\rho_*, 0)$ of (1.1) for sufficiently small g. Furthermore, it was shown that the time-periodic solution is stable under sufficiently small initial perturbations and that the perturbation $(\rho(t) - \rho_{per}(t), v(t) - v_{per}(t))$ satisfies

$$\|(\rho(t) - \rho_{per}(t), v(t) - v_{per}(t))\|_{L^2} \leq C(1+t)^{-\frac{n}{4}}. \tag{1.3}$$

We will show the existence of a time-periodic solution $(\rho_{per}(t), v_{per}(t))$ for $n \geq 3$, if the external force g satisfies the condition $g(-x,t) = -g(x,t)$ $(x \in \mathbb{R}^n,\ t \in \mathbb{R})$ and g is small enough in some weighted Sobolev space. In addition, we will prove that the time-periodic solution $(\rho_{per}(t), v_{per}(t))$ is stable under sufficiently small initial perturbations and that the perturbation $(\rho(t) - \rho_{per}(t), v(t) - v_{per}(t))$ satisfies the decay estimate (1.3).

The results of this talk were obtained in a joint work with Kazuyuki Tsuda (Kyushu University).

References

[1] H. Ma, S. Ukai and T. Yang, Time periodic solutions of compressible Navier-Stokes equations, J. Differential Equations, **248** (2010), pp. 2275–2293.