Periodic Orbits of Hamiltonian Systems: Beyond the Conley Conjecture

Viktor Ginzburg
University of California, Santa Cruz, USA

Abstract: How few periodic orbits can the Reeb flow have, when the contact form gives rise to the standard contact structure on a sphere? Can it have just one closed orbit? In this talk we discuss how methods from Hamiltonian dynamics, originally developed for the proof of the Conley conjecture, translate to the realm of Reeb flows to answer or at least to shed some light on these kinds of questions.

In particular, we prove, drawing from a joint work of the speaker with Hein, Hryniewicz and Macarini, that the existence of one simple closed Reeb orbit of a particular type (a symplectically degenerate maximum) forces the Reeb flow to have infinitely many periodic orbits. We use this result to give a different proof of a recent theorem of Cristofaro-Gardiner and Hutchings asserting that every Reeb flow on the standard contact three-sphere has at least two periodic orbits. (This approach together with several other ingredients leads to a more or less purely symplectic proof of the existence of infinitely many geodesics on the two-sphere.) We also discuss the effect of hyperbolic fixed points on the dynamics of Hamiltonian diffeomorphisms following a recent work of the speaker and Gürel.