

A New Asymptotic Perturbation Theory and Applications to Massless Quantum Fields

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Abstract

Let H_0 and H_I be a self-adjoint and a symmetric operator on a complex Hilbert space, respectively, and suppose that H_0 is bounded below and the infimum E_0 of the spectrum of H_0 is a simple eigenvalue of H_0 which is *not necessarily isolated*. In this paper, we present a new asymptotic perturbation theory for an eigenvalue $E(\lambda)$ of the operator $H(\lambda) := H_0 + \lambda H_I$ ($\lambda \in \mathbf{R} \setminus \{0\}$) satisfying $\lim_{\lambda \rightarrow 0} E(\lambda) = E_0$. The point of the theory is in that it covers also the case where E_0 is a non-isolated eigenvalue of H_0 . Under a suitable set of assumptions, we derive an asymptotic expansion of $E(\lambda)$ up to an arbitrary finite order of λ as $\lambda \rightarrow 0$. We apply the theory to a massless quantum field model.