Milnor fibers of real line arrangements

Masahiko Yoshinaga
Hokkaido University, Japan
email: yoshinaga@math.sci.hokudai.ac.jp

Let $Q(x, y, z) \in \mathbb{C}[x, y, z]$ be a homogeneous polynomial of degree $n + 1$. The variety $F = Q^{-1}(1) \subset \mathbb{C}^3$ is called the Milnor fiber of the hypersurface $X = Q^{-1}(0) \subset \mathbb{C}^3$. The Milnor fiber F is preserved by the scalar multiplication $(x, y, z) \longmapsto (\zeta x, \zeta y, \zeta z)$, where $\zeta = e^{2\pi i/(n+1)}$. It induces an automorphism $\rho : F \longrightarrow F$, so called the Monodromy automorphism. Obviously $\rho^{n+1} = id$. The homology $H_1(F, \mathbb{C})$ equipped with the automorphism induced by the monodromy ρ is an important object. Indeed it is related to several topological invariants (e.g., Local system homology groups, Alexander polynomial of the fundamental group, the number of certain plane curves passing through the prescribed points, and so on).

In this talk, we will discuss the case that $Q = \prod_{i=1}^{n+1} \alpha_i$ splits into the product of linear forms of real coefficients. Then $Q^{-1}(0) = \bigcup_{i=1}^{n+1} H_i$ is a line arrangement in the projective plane $\mathbb{RP}^2$. We will give a new algorithm which computes the monodromy eigen spaces of $H_1(F, \mathbb{C})$ in terms of real and combinatorial structures of chambers. We also give a new upper bound of the dimension of the eigen spaces and several conjectures.

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