Some topological problems and computational methods in the theory of braids and related groups

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The topological theory of Braid groups has been developed in connection with several important subjects, including homotopy theory and singularity theory. From the point of view of cohomological calculations, the theory can be developed in a very similar way for the whole class of groups usually called *Artin groups*. Each of these groups has some (finite or infinite) Coxeter group as quotient, which acts freely over a *configuration space* which is the complement to an *hyperplane arrangement*. The *orbit space* of this action has the original Artin group as its fundamental group.

Many calculations has been performed, starting from the 70's for the trivial cohomology of the braid group and of the Artin groups of finite type.

The explicit construction of CW-complexes over which the orbit space contracts give algebraic complexes computing the twisted cohomology of these groups. In particular, many calculations where produced for abelian local systems over the module of Laurent polynomial. This is a very important module, whose cohomology is strictly related to the (trivial) cohomology of the associated *Milnor fiber*.

We give here some new way to do computations, especially related to typical (even if not old) methods in Combinatorics. We make particular use of some variation of the so called Discrete (or Combinatorial) Morse Theory and we show how the twisted cohomology of the Artin groups can be unexpectedly related to the cohomology of certain graph complexes, which can be computed by using purely combinatorial methods.

The introduction of an interesting class of *weighted sheaves over posets*, being an interesting object by itself, constitutes the bridge between the cohomological theory of Artin groups and combinatorics.

We introduce a spectral sequence associated to such sheaves and we show how discrete (algebraic) Morse theory fits into this theory to allow explicit computations. In particular, besides the case of braid groups, we give explicit cohomology for other groups, including many affine type Artin groups.