Fast diffusion and geometry of domain

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We consider two fast diffusion equations $\partial_t u = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ and $\partial_t u = \Delta u^m$, where $1 < p < 2$ and $0 < m < 1$. Let Ω be a domain in $\mathbb{R}^N$ with $N \geq 2$, and let $u = u(x,t)$ be the solution of either the initial-boundary value problem over Ω, where the initial value equals zero and the boundary value equals 1, or the Cauchy problem where the initial datum is the characteristic function of the set $\mathbb{R}^N \setminus \Omega$. Choose an open ball B in Ω whose closure intersects $\partial\Omega$ only at one point, and let $\alpha > \frac{(N+1)(2-p)}{2p}$ or $\alpha > \frac{(N+1)(1-m)}{4}$. Then, we derive asymptotic estimates for the integral of u^α over B for short times in terms of principal curvatures of $\partial\Omega$ at the point, which tells us about the interaction between fast diffusion and geometry of domain. These kinds of research, which were motivated by C. Cortázar, M. Del Pino and M. Elgueta [CDE], have been done in [MS1, MS2] for the case where $p \geq 2$ or $m \geq 1$. Here in [S] we take into account that the short time behavior of the solutions u is described by the boundary blow-up solutions studied in [BM, M].

References

[BM] C. Bandle and M. Marcus, *Large solutions of semilinear elliptic equations: existence, uniqueness and asymptotic behaviour*, J. Analyse Math. **58** (1992), 9–24.

[CDE] C. Cortázar, M. Del Pino and M. Elgueta, *On the short-time behavior of the free boundary of a porous medium equation*, Duke Math. J. **87** (1997), 133–149.

[MS1] R. Magnanini and S. Sakaguchi, *Interaction between degenerate diffusion and shape of domain*, Proceedings Royal Soc. Edinburgh, Section A **137** (2007), 373–388.

[MS2] R. Magnanini and S. Sakaguchi, *Interaction between nonlinear diffusion and geometry of domain*, J. Differential Equations **252** (2012), 236–257.

[M] J. Matero, *Quasilinear elliptic equations with boundary blow-up*, J. Analyse Math. **69** (1996), 229–247.

[S] S. Sakaguchi, *Interaction between fast diffusion and geometry of domain*, in preparation.