Anisotropic models: analysis of flat regions of solutions

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Anisotropic systems in class of singular parabolic equations generates unusual structures of solutions. One of the most spectacular phenomena are flat regions of solutions, called by the theory: *facets*. Such effects are consequences of the very high singularity of the nonlinear elliptic operator. If the system is anisotropic then the shape of the facets is determined by the anisotropy. From that reason such models find naturally a place in the theory of crystal growth and image processing.

The goal of my talk is to present some current results concerning model problems. I plan to consider the mono-dimensional system

$$u_t - \partial_x(L(u_x)) = 0$$

on an interval with Dirichlet boundary conditions. Fundamental examples are

$$L_0(p) = \operatorname{sgn} p \quad \text{and} \quad L_1(p) = p + \operatorname{sgn} p.$$

For such systems we are able to construct a complete theory explaining the qualitative features of solutions.

The second part is dedicated to two dimensional systems. There are distinguished two equations:

$$u_t - \partial_{x_1}(\operatorname{sgn} u_{x_1}) - \partial_{x_2}(\operatorname{sgn} u_{x_2}) = 0,$$

$$u_t - \partial_{x_1}(\operatorname{sgn} u_{x_1}) - \partial_{x_2}(\operatorname{sgn} u_{x_2}) - \Delta u = 0.$$

Partial analytical results show that the extra linear diffusion in the second equation does not change the main qualitative properties which are characteristic for the first equation. Here we are able to observe not only facets, but also ruled surfaces.

My talk is based on joint results with Piotr Rybka (Warszawa), Monika Muszkieta (Wroclaw) and Karolina Kielak (Warszawa).

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