The tacnode Riemann-Hilbert problem

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Painlevé transcendents arise as special functions in an increasing number of models in mathematical physics and probability theory. A basic example is the appearance of Painlevé II in the description of the Tracy-Widom distribution for the largest eigenvalue of a random matrix.

The tacnode is a critical phenomenon for non-intersecting one-dimensional Brownian motions with prescribed starting and ending points. One can adjust parameters so that the Brownian motions fill out two tangent ellipses in the time-space plane. Local correlations at the tacnode were recently described in two different ways, namely on the one hand with Airy resolvent functions [1], [4], [5], and on the other hand with a Riemann Hilbert problem [3]. The connection between the two sets of formulas was found by Delvaux [2] and depends on an explicit solution of the tacnode Riemann-Hilbert problem in terms of functions related to Painlevé II, see [6].

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