Using persistence to quantify vortex significance

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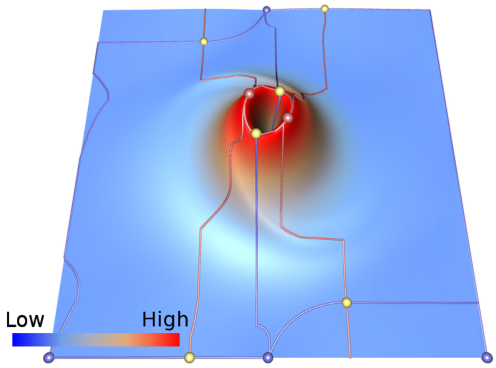
The extraction of vortices plays a fundamental role in the analysis of fluid flows. They are one of the main structures that are relevant for different engineering tasks such as the construction of airfoils or streamlined cars. Unfortunately, the number of vortices rises with the complexity of the flow. Anyhow, not all vortices are equally important. Their influence on the flow field is related to their strength, i.e., the energy transported by the vortex. It is therefore helpful to discriminate between individual vortices by assigning an importance measure to theses structures.

The first step to extract vortex cores is to determine a set of feature points. Typically, these points are defined as extremal structures of a certain scalar feature identifier. Different feature identifiers have been presented in the past. Well-known examples are the Okubo-Weiss criterion (Okubo 1970, Weiss 1991), vorticity, the λ_2 criterion (Jeong and Hussain 1995). In this work, we consider the extraction of vortices in two-dimensional time-dependent flow fields. Here, we want to use the scalar quantity not only as a feature identifier but also as a measure that resembles the importance or strength of a vortex. We use the acceleration magnitude of the flow field. The acceleration is a fundamental part of the Navier-Stokes equations. In the right hand figure, the acceleration magnitude of a Lundgren vortex is shown. It can be seen that the vortex core is surrounded by a particularly pronounced ridge. The height of this ridge varies with the strength of the vortex. It resembles the ability of the vortex to attract particles towards the vortex core.



Using the fact that the strength of the vortex is related to the height of the surrounding ridge in the acceleration magnitude, we now have to find a robust approach to quantify this height difference. To do so, persistent homology as proposed by Edelsbrunner [1] can be used. Loosely speaking, persistence measures the robustness of a critical point against perturbations of the scalar values within its vicinity. In the two-dimensional case, the persistence of a minimum is determined by the height difference to its connected saddle. In the case of the acceleration magnitude, the vortex core is given by a minimum in the scalar field topology. The connected saddle point lies on the ridge surrounding the minimum. Thus, persistence indeed measures the strength of the vortex.

In addition to analyzing slices of a two-dimensional time-dependent data set, the importance measure given by persistence can be extended to the temporal domain by incorporating the lifetime of a vortex. Integrating the persistence value along a tracked vortex core line results in a measure that marks spatially important and long-living vortices.

The extraction of persistence can be done in a combinatorial setting that also includes the extraction of the minima of the acceleration magnitude. We therefore have a unified framework to extract the vortex cores and their strength. The tracking can also be accomplished in the same combinatorial context.

More information about the extraction of vortices using the acceleration magnitude and tracking vortex core in time-dependent flow fields can be found in [2].

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Bibliography: [1] H. Edelsbrunner, J. Harer, and A. Zomorodian: Hierarchical Morse Complexes for Piecewise Linear 2-manifolds. In Proceedings Symposium on Computational Geometry 2001, pages 70-79.
[2] J. Kasten: Lagrangian Feature Extraction in Two-dimensional Unsteady Flows. PhD thesis, FU Berlin, 2012.