Mickelsson's twisted K-theory invariant

Kiyonori Gomi
Department of Mathematical Sciences, Shinshu University

Twisted K-theory, being a topological K-theory with certain local coefficients, was invented by Donovan-Karoubi and also by Rosenberg. This notion is originally applied to a generalization of the Thom isomorphism theorem, and recently to mathematical physics. For example, in the context of string theory, twisted K-theory is thought of as home of Ramond-Ramond charges of D-branes with background B-fields.

Mickelsson's invariant is an invariant of some odd twisted K-classes on 3-manifolds [1]. The point of this invariant is that it detects some torsion elements, in contrast to the Chern character, which detects all the non-torsion elements but no non-torsion elements.

The theme of my talk is a reformulation of Mickelsson's invariant: For its account, we denote by $K^1_P(M)$ the odd twisted K-group of a manifold M, where P is a principal bundle whose structure group is the projective unitary group $PU(H) = U(H)/U(1)$ of a separable infinite dimensional Hilbert space H. Such a principal bundle is classified by a cohomology class $h(P) \in H^3(M, \mathbb{Z})$. Then there is a simplest odd twisted K-theory invariant, namely, a natural homomorphism $\mu_1 : K^1_P(M) \to H^1(M, \mathbb{Z})$.

Theorem 1. *For any principal $PU(H)$-bundle P on a manifold M, there is a natural homomorphism*

$$\mu_3 : \ \mathrm{Ker}\mu_1 \longrightarrow H^3(M, \mathbb{Z})/(\mathrm{Tor} + h(P) \cup H^0(M, \mathbb{Z})),$$

where Tor *is the torsion subgroup. The homomorphism recovers original Mickelsson's invariant if M is compact, oriented, connected and 3-dimensional.*

The homomorphism μ_3, which is constructed by using a Čech-de Rham cocycle, factors through a homomorphism in computing the Atiyah-Hirzebruch spectral sequence. By the help of this factorization and a computation of μ_3, we can reproduce the known result that $K^1_P(SU(3)) \cong \mathbb{Z}/h$ in the case that $h = h(P) \in H^3(SU(3), \mathbb{Z}) \cong \mathbb{Z}$ is odd.

References

[1] J. Mickelsson, *Twisted K theory invariants*. Lett. Math. Phys. 71 (2005), no. 2, 109–121.