Energy, pseudomomentum and Stokes drift of inertial waves and their application to stability of a columnar vortex

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Abstract

A steady Euler flow of an inviscid incompressible fluid is characterized as an extremum of the total kinetic energy (=the Hamiltonian) with respect to perturbations constrained to an isovortical sheet (=coadjoint orbits). We exploit the criticality in the Hamiltonian to calculate the energy of the inertial waves or Kelvin waves, three-dimensional waves on a steady vortical flow [1, 2, 3], and, as a by-product, to calculate the mean flow of second order in amplitude, induced by nonlinear interaction of waves with themselves [4]. We pursue the relation of this mean flow with the pseudomomentum and the Stokes drift [5].

We then apply these formulas to the linear and weakly nonlinear stability of a rotating flow confined in a cylinder of elliptic cross-section. The linear instability is known as the Moore-Saffman-Tsai-Widnall (MSTW) instability, and its characteristics as parametric resonance between a pair of Kelvin waves is well captured from the viewpoint of Krein's theory of Hamiltonian spectra [1]. The wave-induced mean flow is indispensable for proceeding to the weakly nonlinear stage. A hybrid method of combining the Eulerian and the Lagrangian approaches is developed to deduce the amplitude equations to third order [6, 7, 5]. By an appropriate normalization of dependent variables, the resulting amplitude equations are made into Hamiltonian form [5]. The linear instability is connected to the Hamiltonian pitchfork and Hopf bifurcations.

References

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