Convergence for a 2D elliptic problem with large exponent in nonlinearity

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In this talk, we are concerned with the problem

$$(E_p) \quad \begin{cases} -\Delta u = K(x)u^p & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a smooth bounded domain in $\mathbb{R}^2$, $K \in C^1(\overline{\Omega})$, $\inf_{\overline{\Omega}} K > 0$ is a given positive weight function, and $p > 1$ is a nonlinear exponent.

Let $\{u_p\}$ be a sequence of solutions to (E_p), not necessarily least energy ones. The main purpose of this talk is to investigate the asymptotic behavior of general solutions u_p when the nonlinear exponent p gets large.

Let $\{p_n\}$ be a sequence of exponents with $p_n > 1, p_n \to +\infty$, and $\{u_{p_n}\}$ be a solution sequence of (E_p) for $p = p_n$. In this talk, we prove that along a subsequence (again denoted by $\{p_n\}$), mass quantization occurs in the sense that

$$p_n \int_\Omega K(x) u_{p_n}^{p_n} dx \to 8\pi\sqrt{e}N, \quad N \in \mathbb{N} \cup \{+\infty\}.$$

Furthermore, we have the entire blow-up if $N = +\infty$, or, N-points concentration holds if $N \in \mathbb{N}$, in the sense that there exists a set of N points $\mathcal{S} = \{x_1, \cdots, x_N\} \subset \Omega$ such that, as $p_n \to \infty$,

$$\lim_{n\to\infty} \sup_{B_r(x_i)} u_{p_n} = \lim_{n\to\infty} \|u_{p_n}\|_{L^\infty(\Omega)} = \sqrt{e} \quad (r > 0 \text{ small}),$$

$$p_n K(x) u_{p_n}^{p_n} \stackrel{*}{\rightharpoonup} 8\pi\sqrt{e} \sum_{i=1}^N \delta_{x_i} \text{ in the sense of measures on } \overline{\Omega},$$

$$p_n u_{p_n} \to 8\pi\sqrt{e} \sum_{i=1}^N G(\cdot, x_i) \text{ in } C^1_{loc}(\overline{\Omega} \setminus \mathcal{S}).$$

Also we obtain a characterization of each concentration point as a critical point of some function defined by the Green function and the coefficient function K.

These results are obtained by using ideas and techniques from the recent paper by S. Santra and J.C. Wei (J. d'Analyse Math. 2011) with suitable modifications.