

# KNOTTED VORTEX TUBES IN STEADY SOLUTIONS TO THE EULER EQUATION

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The motion of particles in an ideal fluid in  $\mathbb{R}^3$  is described by its velocity field  $u(x, t)$ , which satisfies the Euler equation

$$\partial_t u + (u \cdot \nabla)u = -\nabla P, \quad \operatorname{div} u = 0$$

for some pressure function  $P(x, t)$ . The trajectories of the vorticity  $\omega(x, t)$  for fixed  $t$  are usually called *vortex lines*. A solution  $u$  to the Euler equation is called *steady* when it does not depend on time.

A domain in  $\mathbb{R}^3$  that is the union of vortex lines and whose boundary is an embedded torus is a (closed) *vortex tube*. The analysis of *thin* vortex tubes for solutions to the Euler equation has attracted considerable attention. A long-standing problem in this direction is Lord Kelvin's conjecture that knotted and linked thin vortex tubes can arise in steady solutions to the Euler equation. This is basically a question on the existence of knotted invariant tori in steady solutions of the Euler equation.

In this talk we will show the existence of steady solutions to the Euler equation in  $\mathbb{R}^3$  having thin vortex tubes of any link and knot type. The steady solutions we construct are *Beltrami fields* (that is, they satisfy the equation  $\operatorname{curl} u = \lambda u$  in  $\mathbb{R}^3$  for some nonzero real constant  $\lambda$ ) and fall off at infinity as  $|u(x)| < C/|x|$ .

The structure of the vortex lines inside each vortex tube is extremely rich. In the first place, it can be shown that the cores of the thin tubes are knotted vortex lines, so previous results on the existence of knotted trajectories are recovered [1]. In the second place, the vortex tubes we construct are *stable*, in a sense that we will make precise. This is related to the existence of infinitely many invariant tori in the interior of the vortex tube. The proof of these results combines fine estimates for the Beltrami equation in a thin tube with ideas from KAM theory.

## REFERENCES

- [1] A. Enciso, D. Peralta-Salas, Knots and links in steady solutions of the Euler equation, *Ann. of Math.* 175 (2012) 345–367.
- [2] A. Enciso, D. Peralta-Salas, Existence of knotted vortex tubes in steady Euler flow, [arXiv:1210.6271](https://arxiv.org/abs/1210.6271).

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