On the relationship between the Painlevé equations and semi-classical orthogonal polynomials

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Abstract

In this talk I will be concerned with the relationship between the Painlevé equations and orthogonal polynomials with respect to semi-classical weights. It is well-known that orthogonal polynomials satisfy a three-term recurrence relation and for some semi-classical weights, these coefficients in the recurrence relation can be expressed in terms of solutions of a Painlevé equation. I will show that the coefficients in these recurrence relations can be expressed in terms of Wronskians of special functions which arise in the description of classical solutions of Painlevé equations.

Specifically I shall discuss orthogonal polynomials with respect to a semi-classical Laguerre weight and variations of the Freud weight. For these orthogonal polynomials, the coefficients in the three-term recurrence relations can be expressed in terms of Wronskians of parabolic cylinder functions that arise in connection with special function solutions of the fourth Painlevé equation.

I shall also discuss semi-classical generalizations of the Charlier and Meixner polynomials, which are discrete orthogonal polynomials whose recurrence coefficients can be expressed in terms of Wronskians of modified Bessel functions and confluent hypergeometric functions (equivalently Kummer or Whittaker functions), respectively. These Wronskians arise in the description of special function solutions of the third and fifth Painlevé equations.