

Virtual neighborhood technique for pseudo-holomorphic spheres

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Abstract:

The main analytical difficulty in defining the Gromov-Witten invariants for general symplectic manifolds is the failure of the transversality of the compactified moduli space of pseudo-holomorphic curves. The foundation to resolve this issue is to construct a virtual fundamental cycle for the compactified moduli space. For smooth projective varieties, the construction of this virtual fundamental cycle was carried out by Li-Tian. For general symplectic manifolds, the virtual fundamental cycle was constructed by Fukaya-Ono, Li-Tian, Liu-Tian. Ruan proposed a virtual neighborhood technique as a dual approach using the Euler class of a virtual neighborhood, in which the compactified moduli space is a zero set of a smooth section of a finite dimensional orbifold vector bundles over an open orbifolds.

Further developments in Gromov-Witten theory and its applications require differential structures on these moduli spaces involved. Some of the analytical details have been provided by Ruan, Li-Ruan and Fukaya-Oh-Ohta-Ono. Other methods like the polyfold theory by Hofer-Wysocki-Zehnder are developed to deal with this issue. In a recent joint work with Bohui Chen and Anmin Li, we implement the full machinery of virtual neighborhood technique to the Gromov-Witten theory using virtual manifold/orbifolds developed by Chen-Tian. This provides an alternative approach to establish differentiable structure on moduli spaces arising from the Gromov-Witten invariants. In this short talk, I explain how this method can be applied to the genus 0 Gromov-Witten theory by resolving the analytical issue of the non-differentiable $PSL(2, \mathbb{C})$ -action.