MINIMAL AND CONSTANT MEAN CURVATURE SURFACES IN THE THREE-SPHERE: BRENDLE'S PROOF OF THE LAWSON CONJECTURE

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Minimal surfaces (surfaces of least area) and related objects such as constant mean curvature surfaces have formed a substantial part of the development of the field of geometric analysis. Not only are they beautiful objects with a rich theory spanning partial differential equations, calculus of variations and complex analysis, but they have been employed to great effect in efforts to understand the topology and geometry of manifolds, particularly in three dimensions.

Recently there has been some spectacular progress in this field, answering some old questions about the nature of these surfaces in the simplest possible three-dimensional manifold: The three-dimensional sphere. Simon Brendle gave a remarkable proof [B] of a conjecture of H. Blaine Lawson [L1] dating back to the 1970s, that the only *embedded* minimal torus in the three-sphere (up to congruence) is the Clifford torus, which is the product $S^1(1/\sqrt{2}) \times S^1(1/\sqrt{2}) \subset \mathbb{R}^2 \times \mathbb{R}^2$. Subsequently Haizhong Li and I used related ideas to give a classification of constant mean curvature embedded tori in the three-sphere, proving in particular a conjecture of Pinkall and Sterling [PS] that all such surfaces should be axially symmetric.

In this talk I will discuss briefly some of the background to these conjectures, including the beautiful construction of higher genus embedded minimal surfaces due to Lawson [L2], and the construction of large numbers of *immersed* minimal tori using integrable systems [PS]. Then I will describe one of the new ingredients employed by Brendle, which is a geometric estimate originating from some recent work of mine on mean curvature flow [A]. Finally, I will explain how this is used in the proof. If time allows I will also describe the situation for constant mean curvature tori and some more recent related work on Weingarten tori.

References

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