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Linear Weingarten hypersurfaces in locally symmetric manifolds

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Abstract. In this paper, we discuss with n-dimensional complete orientable linear Weingarten hypersurface in locally symmetric manifold and obtain some rigidity results.

Key words: Linear Weingarten hypersurfaces; locally symmetric manifolds, δ-pinching.

1. Introduction

Recently, many researchers studied the minimal hypersurfaces or hypersurfaces with constant mean (or scalar) curvature in the locally symmetric manifolds and the δ-pinched manifolds, and obtained many rigidity results about these hypersurfaces ([4], [10], [11] and the references therein). In this paper, we modify Cheng-Yau's technique to complete linear Weingarten hypersurfaces in locally symmetric manifolds and prove some rigidity theorems under the hypothesis of the mean curvature and the normalized scalar curvature being linearly related. More precisely, we have

Theorem 1.1 *Let N^{n+1} $(n \geq 3)$ be a locally symmetric manifold satisfying $1/2 < \delta \leq K_N \leq 1$ and $K_{n+1in+1i} = c_0$. Let M^n be an n-dimensional complete orientable linear Weingarten hypersurface of N^{n+1}, such that $r = aH + b$ with $b > 1$. If H attains its maximum on M^n and $S \leq 2\sqrt{n-1}(2\delta - c_0)$, then either M^n is totally umbilical hypersurface or M^n has two distinct constant principal curvatures, one of which is simple.*

When $\delta = c_0 = 1$, N^{n+1} is a unit sphere $S^{n+1}(1)$, so we have the following corollary by the theorem 1.1(2i) of [9].

Corollary 1.2 *Let M^n be an n-dimensional complete orientable linear Weingarten hypersurface of $S^{n+1}(1)$, such that $r = aH + b$ with $b > 1$. If H attains its maximum on M^n and $S \leq 2\sqrt{n-1}$, then either M^n is*

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totally umbilical hypersurface or M^n is isometric to a Riemannian product $S^{n-1}(c) \times S^1(\sqrt{1-c^2})$.

Theorem 1.3 *Let N^{n+1} $(n \geq 3)$ be a locally symmetric Einstein manifold satisfying $1/2 < \delta \leq K_N \leq 1$ and $K_{n+1in+1i} = c_0$. Let M^n be an n-dimensional complete noncompact orientable linear Weingarten hypersurface of N^{n+1}, such that $r = aH + b$ with $b > 1$. If $S \leq 2\sqrt{n-1}(2\delta - c_0)$ and $|\nabla H| \in \mathcal{L}^1(M)$, then either M^n is totally umbilical hypersurface or M^n has two distinct constant principal curvatures, one of which is simple.*

2. Preliminaries

Let N^{n+1} be a locally symmetric manifold and M^n be an n-dimensional complete and orientable hpersurface in N^{n+1}. For any $p \in M$, we choose a local orthonormal frame $e_1, \ldots, e_{n+1}$ in N^{n+1} around p such that $e_1, \ldots, e_n$ are tangent to M^n. Let $\omega_1, \ldots, \omega_{n+1}$ be the corresponding dual coframe. We use the following standard convention for indices:

$$1 \leq A, B, C, \cdots \leq n+1, \qquad 1 \leq i, j, k, \cdots \leq n.$$

The structure equations of N^{n+1} are given by

$$d\omega_A = -\sum_B \omega_{AB} \wedge \omega_B, \quad \omega_{AB} + \omega_{BA} = 0, \tag{2.1}$$

$$d\omega_{AB} = -\sum_C \omega_{AC} \wedge \omega_{CB} + \frac{1}{2}\sum_{C,D} \varepsilon_C \varepsilon_D K_{ABCD} \omega_C \wedge \omega_D, \tag{2.2}$$

where K_{ABCD} are the components of the curvature tensor of N^{n+1}.

Restricting these forms to M^n, we have $\omega_{n+1} = 0$. Since $0 = d\omega_{n+1} = -\sum_i \omega_{n+1i} \wedge \omega_i$, from Cartan lemma, we can write

$$\omega_{n+1i} = \sum_j h_{ij} \omega_j, \quad h_{ij} = h_{ji}. \tag{2.3}$$

Let $B = \sum_{i,j} h_{ij} \omega_i \omega_j e_{n+1}$ be the second fundamental form. We will denote by $h = (1/n) \sum_i h_{ii} e_{n+1}$ and by $H = |h| = (1/n) \sum_i h_{ii}$ the mean curvature vector and the mean curvature of M^n, respectively.

The structure equations of M^n are

$$d\omega_i = -\sum_{j=1}^{n} \omega_{ij} \wedge \omega_j, \quad \omega_{ij} + \omega_{ji} = 0, \tag{2.4}$$

$$d\omega_{ij} = -\sum_{k=1}^{n} \omega_{ik} \wedge \omega_{kj} + \frac{1}{2} \sum_{k,l=1}^{n} R_{ijkl}\omega_k \wedge \omega_l. \tag{2.5}$$

The Gauss equations are

$$R_{ijkl} = K_{ijkl} + (h_{ik}h_{jl} - h_{il}h_{jk}), \tag{2.6}$$

$$n(n-1)r = \sum_{i,j} K_{ijij} + n^2H^2 - S, \tag{2.7}$$

where r is the normalized scalar curvature of M^n and $S = \sum_{i,j} h_{ij}^2$ is the norm square of the second fundamental form of M^n.

The Codazzi and Ricci equations are

$$h_{ijk} - h_{ikj} = -K_{n+1ijk}, \tag{2.8}$$

$$K_{n+1ijkl} = K_{n+1in+1k}h_{jl} + K_{n+1ijn+1}h_{kl} - \sum_{m} K_{mijk}h_{ml}, \tag{2.9}$$

where the covariant derivative of h_{ij} is defined by

$$\sum_k h_{ijk}\omega_k = dh_{ij} - \sum_k h_{kj}\omega_{ki} - \sum_k h_{ik}\omega_{kj}. \tag{2.10}$$

Similarly, the components h_{ijkl} of the second derivative $\nabla^2 h$ are given by

$$\sum_l h_{ijkl}\omega_l = dh_{ijk} - \sum_l h_{ljk}\omega_{li} - \sum_l h_{ilk}\omega_{lj} - \sum_l h_{ijl}\omega_{lk}. \tag{2.11}$$

The Laplacian $\triangle h_{ij}$ of h_{ij} is defined by

$$\triangle h_{ij} = \sum_k h_{ijkk}.$$

By a simple and direct calculation, we have

$$
\begin{aligned}
\triangle h_{ij} &= \sum_k \big[(h_{ijkk} - h_{ikjk}) + (h_{ikjk} - h_{ikkj}) + (h_{ikkj} - h_{kkij}) + h_{kkij}\big] \\
&= \sum_k K_{n+1ikjk} + \sum_{k,m}(h_{mi}R_{mkjk} + h_{mk}R_{mijk}) \\
&\quad + \sum_k K_{n+1kkij} + \sum_k h_{kkij} \\
&= (nH)_{ij} + nHK_{n+1in+1j} - \sum_k h_{ij}K_{n+1kn+1k} + nH\sum_k h_{ik}h_{kj} \\
&\quad - Sh_{ij} + \sum_k \big[h_{mi}K_{mkjk} + h_{mj}K_{mkik} + 2h_{km}K_{mijk}\big]. \qquad (2.12)
\end{aligned}
$$

We choose a local frame of orthonormal vectors fields $\{e_i\}$ such that at arbitrary fixed point p of M^n

$$h_{ij} = \lambda_i \delta_{ij}, \qquad (2.13)$$

then it follows, at p, that

$$
\begin{aligned}
\frac{1}{2}\triangle S = \frac{1}{2}\sum_{i,j}\triangle h_{ij}^2 &= \sum_{i,j,k} h_{ijk}^2 + \sum_{i,j} h_{ij}\triangle h_{ij} \\
&= \sum_{i,j,k} h_{ijk}^2 + \sum_i \lambda_i (nH)_{ii} - S^2 + nH\sum_i \lambda_i^3 \\
&\quad + nH\sum_i \lambda_i K_{n+1in+1i} - S\sum_i K_{n+1in+1i} \\
&\quad + \sum_{i,j}(\lambda_i - \lambda_j)^2 K_{ijij}. \qquad (2.14)
\end{aligned}
$$

Set $\phi_{ij} = h_{ij} - H\delta_{ij}$, it is easy to check that ϕ is traceless and

$$|\phi|^2 = \sum_{i,j}(\phi_{ij})^2 = S - nH^2, \qquad (2.15)$$

where ϕ denotes the matrix (ϕ_{ij}). Moreover, $|\phi|^2 = S - nH^2 \geq 0$ with equality holds if and only if M^n is totally umbilical.

Lemma 2.1 ([7]) *Let $u_1, u_2, \dots, u_n$ be real numbers such that $\sum_i u_i = 0$ and $\sum_i u_i^2 = \beta$. Then*

$$-\frac{n-2}{\sqrt{n(n-1)}}\beta^3 \leq \sum_i u_i^3 \leq \frac{n-2}{\sqrt{n(n-1)}}\beta^3,$$

and equality holds if and only if at least $n-1$ of $u_i's$ are equal.

Lemma 2.2 *Let N^{n+1} be a locally symmetric manifold satisfying $1/2 < \delta \leq K_N \leq 1$ and M^n be an n-dimensional complete orientable hypersurface of N^{n+1} with $r = aH + b, a, b \in \mathbb{R}$ and $(n-1)a^2 + 4n(b-1) \geq 0$. Then we have*

$$\sum_{i,j,k} h_{ijk}^2 \geq n^2 |\nabla H|^2, \tag{2.16}$$

and equality holds if and only if $|\nabla H|^2 = 0$ or $4n^2 S = (2n^2 H - n(n-1)a)^2$. Moreover, if $(n-1)a^2 + 4n(b-1) > 0$ and the equality holds in (2.16) on M^n, then H is constant on M^n.

Proof. From Gauss equation, we have

$$\begin{aligned} S &= \sum_{i,j} K_{ijij} + n^2 H^2 - n(n-1)r \\ &= \sum_{i,j} K_{ijij} + n^2 H^2 - n(n-1)(aH + b). \end{aligned} \tag{2.17}$$

Since N^{n+1} is locally symmetric, taking the covariant derivative of the above equation, we have

$$2 \sum_{i,j} h_{ij} h_{ijk} = 2n^2 H H_k - n(n-1) a H_k.$$

Therefore,

$$4S \sum_{i,j,k} h_{ijk}^2 \geq 4 \sum_k \left(\sum_{i,j} h_{ij} h_{ijk} \right)^2 = (2n^2 H - n(n-1)a)^2 |\nabla H|^2. \tag{2.18}$$

We know from $0 < \delta \leq K_{ijij} \leq 1$ that

$$\begin{aligned}
&(2n^2H - n(n-1)a)^2 - 4n^2S \\
&\quad = 4n^4H^2 + n^2(n-1)^2a^2 - 4n^3(n-1)aH \\
&\qquad - 4n^2\Big(\sum_{i,j} K_{ijij} + n^2H^2 - n(n-1)(aH+b)\Big) \\
&\quad \geq 4n^4H^2 + n^2(n-1)^2a^2 - 4n^3(n-1)aH \\
&\qquad - 4n^2\big(n(n-1) + n^2H^2 - n(n-1)(aH+b)\big) \\
&\quad = n^2(n-1)^2a^2 + 4n^3(n-1)(b-1) \\
&\quad = n^2(n-1)\big((n-1)a^2 + 4n(b-1)\big) \geq 0. \qquad (2.19)
\end{aligned}$$

It follows (2.18) and (2.19) that

$$4S\sum_{i,j,k} h_{ijk}^2 \geq (2n^2H - n(n-1)a)^2|\nabla H|^2 \geq 4n^2S|\nabla H|^2. \qquad (2.20)$$

Thus either $S = 0$ and $\sum_{i,j,k} h_{ijk}^2 = n^2|\nabla H|^2$ or $\sum_{i,j,k} h_{ijk}^2 \geq n^2|\nabla H|^2$.

If $\sum_{i,j,k} h_{ijk}^2 = n^2|\nabla H|^2$, from (2.18) and (2.19), we have

$$\begin{aligned}
0 &\leq n^2(n-1)\big((n-1)a^2 + 4n(b-1)\big)|\nabla H|^2 \\
&\leq (2n^2H - n(n-1)a)^2|\nabla H|^2 - 4n^2S|\nabla H|^2 \\
&\leq 4S\sum_{i,j,k} h_{ijk}^2 - 4n^2S|\nabla H|^2 = 4S\Big(\sum_{i,j,k} h_{ijk}^2 - n^2|\nabla H|^2\Big) = 0.
\end{aligned}$$

Then we conclude that $|\nabla H|^2 = 0$ or $4n^2S = (2n^2H - n(n-1)a)^2$.

Moreover, if $(n-1)a^2 + 4n(b-1) > 0$ and $\sum_{i,j,k} h_{ijk}^2 = n^2|\nabla H|^2$, from (2.19) and (2.20), we have $|\nabla H|^2 = 0$ on M^n and, hence, H is constant on M^n. □

Following Cheng-Yau [3], as in [2], we introduce a modified operator L acting on any C^2- function f by

$$L(f) = \sum_{i,j}\Big(\Big(nH - \frac{n-1}{2}a\Big)\delta_{ij} - h_{ij}\Big)f_{ij}, \qquad (2.21)$$

where f_{ij} is given by the following

$$\sum_j f_{ij}\omega_j = df_i + f_j\omega_{ij}.$$

Lemma 2.3 *Let N^{n+1} be a locally symmetric manifold satisfying $1/2 < \delta \le K_N \le 1$ and M be an n-dimensional orientable linear Weingarten hypersurface with $r = aH + b$ in N^{n+1}. If $b > 1$, then L is elliptic.*

Proof. Since $r = aH + b$ and $K_N \le 1$, from Gauss equation (2.7), we get

$$n(n-1)(aH+b) \le n(n-1) + n^2H^2 - S,$$

i.e.

$$S \le n^2H^2 - n(n-1)(b-1) - n(n-1)aH. \tag{2.22}$$

Since $b > 1$, we know that

$$n^2H^2 - n(n-1)aH - S \ge n(n-1)(b-1) > 0. \tag{2.23}$$

Therefore $H \neq 0$. Thus we can assume $H > 0$ on M. So L is elliptic if and only if $nH - ((n-1)/2)a - \lambda_i > 0$ for $i = 1, 2, \ldots, n$, where $\lambda_i' s$ are the principal curvatures of M. From (2.22) we have

$$a \le \frac{1}{n(n-1)H}\big(n^2H^2 - S - n(n-1)(b-1)\big).$$

Consequently, we obtain

$$\begin{aligned}
&nH - \frac{n-1}{2}a - \lambda_i \\
&\quad\ge \frac{1}{2nH}\big(n^2H^2 + S - 2nH\lambda_i + n(n-1)(b-1)\big) \\
&\quad= \frac{1}{2nH}\bigg(\Big(\sum_j \lambda_j\Big)^2 + \sum_j \lambda_j^2 - 2\lambda_i\sum_j \lambda_j + n(n-1)(b-1)\bigg) \\
&\quad= \frac{1}{2nH}\bigg(\Big(\sum_{j\neq i} \lambda_j\Big)^2 + \sum_{j\neq i} \lambda_j^2 + n(n-1)(b-1)\bigg).
\end{aligned}$$

Therefore, since $b > 1$, we conclude that L is an elliptic operator. $\square$

3. The proof of theorems

Firstly, we give the following proposition.

Proposition 3.1 *Let N^{n+1} $(n \geq 3)$ be a locally symmetric manifold satisfying $1/2 < \delta \leq K_N \leq 1$, $K_{n+1in+1i} = c_0$ and M^n be an n-dimensional complete orientable hypersurface of N^{n+1} with $r = aH + b, a, b \in \mathbb{R}$ and $(n-1)a^2 + 4n(b-1) \geq 0$. Then the following inequality holds*

$$L(nH) \geq -\frac{n}{2\sqrt{n-1}}\big[S - 2\sqrt{n-1}(2\delta - c_0)\big]|\phi|^2. \tag{3.1}$$

Proof. From (2.21) we have

$$\begin{aligned}
L(nH) &= \sum_{i,j}\left(\left(nH - \frac{1}{2}(n-1)a\right)\delta_{ij} - h_{ij}^{n+1}\right)(nH)_{ij} \\
&= \left(nH - \frac{1}{2}(n-1)a\right)\triangle(nH) - \sum_{i,j} h_{ij}^{n+1}(nH)_{ij} \\
&= \left(nH - \frac{1}{2}(n-1)a\right)\triangle\left(nH - \frac{1}{2}(n-1)a\right) - \sum_{i,j} h_{ij}^{n+1}(nH)_{ij} \\
&= \frac{1}{2}\triangle\left(nH - \frac{1}{2}(n-1)a\right)^2 - \left|\nabla\left(nH - \frac{1}{2}(n-1)a\right)\right|^2 \\
&\quad - \sum_{i,j} h_{ij}^{n+1}(nH)_{ij} \\
&= \frac{1}{2}\triangle\left(nH - \frac{1}{2}(n-1)a\right)^2 - n^2|\nabla H|^2 - \sum_{i,j} h_{ij}^{n+1}(nH)_{ij}.
\end{aligned} \tag{3.2}$$

Since the scalar curvature $\bar{R}$ of locally symmetric manifold is constant. Then, from

$$\bar{R} = 2\sum_i K_{n+1in+1i} + \sum_{i,j} K_{ijij} = 2nc_0 + \sum_{i,j} K_{ijij},$$

we know that $\sum_{i,j} K_{ijij}$ is constant. Therefore, from Gauss equation and $r = aH + b$, we have

$$\begin{aligned}\triangle S &= \triangle\Big(\sum_{i,j} K_{ijij} + n^2H^2 - n(n-1)r\Big)\\ &= \triangle(n^2H^2 - n(n-1)(aH+b))\\ &= \triangle(n^2H^2 - n(n-1)aH)\\ &= \triangle\Big(nH - \frac{1}{2}(n-1)a\Big)^2. \end{aligned}\tag{3.3}$$

From (2.14), (3.2) and (3.3), we get

$$\begin{aligned}L(nH) &= \frac{1}{2}\triangle S - n^2|\nabla H|^2 - \sum_{i,j} h_{ij}^{n+1}(nH)_{ij}\\ &= \sum_{i,j,k} h_{ijk}^2 - n^2|\nabla H|^2 - S^2 + nH\sum_i \lambda_i^3 + \sum_{i,j}(\lambda_i-\lambda_j)^2K_{ijij}\\ &\quad + nH\sum_i \lambda_i K_{n+1in+1i} - S\sum_i K_{n+1in+1i}. \end{aligned}\tag{3.4}$$

On the other hand, putting $\mu_i = \lambda_i - H$, we can obtain

$$\sum_i \mu_i = 0, \quad \sum_i \mu_i^2 = |\phi|^2 = S - nH^2, \quad \sum_i \mu_i^3 = \sum_i \lambda_i^3 - 3HS + 2nH^3.$$

Then, for any $\varepsilon > 0$, we have

$$\begin{aligned}-S^2 + nH\sum_i \lambda_i^3 &= -S^2 + nH\sum_i \mu_i^3 + 3nH^2S - 2n^2H^4\\ &\geq -\frac{n(n-2)}{\sqrt{n(n-1)}}|H||\phi|^3 + nH^2|\phi|^2 - |\phi|^4\\ &\geq -\frac{n-2}{2\sqrt{n-1}}\Big(n\varepsilon H^2 + \frac{1}{\varepsilon}|\phi|^2\Big)|\phi|^2 + nH^2|\phi|^2 - |\phi|^4. \end{aligned}\tag{3.5}$$

When $n \geq 3$, taking $\varepsilon = (n + 2\sqrt{n-1})/(n-2)$ in (3.5), we get

$$-S^2 + nH\sum_i \lambda_i^3 \geq -\frac{n}{2\sqrt{n-1}}(nH^2|\phi|^2 + |\phi|^4) = -\frac{n}{2\sqrt{n-1}}S|\phi|^2. \quad (3.6)$$

Since N is a δ-pinched manifold, we have

$$\sum_{i,j}(\lambda_i - \lambda_j)^2 K_{ijij} \geq \delta \sum_{i,j}(\lambda_i - \lambda_j)^2 = 2n\delta|\phi|^2, \quad (3.7)$$

At the same time, using the curvature condition, we get

$$nH\sum_i \lambda_i K_{n+1in+1i} - S\sum_i K_{n+1in+1i} = nc_0(n^2H^2 - S) = -nc_0|\phi|^2. \quad (3.8)$$

From (3.4), (3.6), (3.7), (3.8) and Lemma 2.2, we see that

$$\begin{aligned} L(nH) &\geq -nc_0|\phi|^2 + 2n\delta|\phi|^2 - \frac{n}{2\sqrt{n-1}}S|\phi|^2 \\ &= -\frac{n}{2\sqrt{n-1}}\big[S - 2\sqrt{n-1}(2\delta - c_0)\big]|\phi|^2. \end{aligned} \quad (3.9)$$

□

Proof of theorem 1.1. From (3.1) and the assumption $S \leq 2\sqrt{n-1}(2\delta - c_0)$, we get

$$L(nH) \geq -\frac{n}{2\sqrt{n-1}}\big[S - 2\sqrt{n-1}(2\delta - c_0)\big]|\phi|^2 \geq 0. \quad (3.10)$$

Since Lemma 2.3 guarantees that L is elliptic and as we are supposing that H attains its maximum on M^n, from (3.10) we conclude that H is constant on M^n. Thus (3.10) become an equality. If $S < 2\sqrt{n-1}(2\delta - c_0)$, then $|\phi|^2 \equiv 0$ and M^n is totally umbilical. If $S = 2\sqrt{n-1}(2\delta - c_0)$, then all the equalities to obtain (3.10) become equalities. Especially the equality in Lemma 2.1 holds, we have that M^n has two distinct principle curvature, one of which is simple. Since H and S are constants, it is easy to know that M^n has two distinct constant principal curvatures, one of them being simple. □

Lemma 3.2 ([1]) *Let X be a smooth vector field on the n-dimensional complete noncompact oriented Riemannian manifold M^n, such that $div_M X$*

does not change sign on M^n. If $|X| \in \mathcal{L}^1(M)$, then $div_M X = 0$.

Proof of theorem 1.3. Firstly, we easily obtain from (2.21) that

$$L(nH) = div_M(P(\nabla H)), \tag{3.11}$$

where $P = (n^2H - (n(n-1)/2)a)I - nA$ and I denotes the identity operator and A denotes the second fundamental form of M^n.

Moreover, since $nH^2 \leq S \leq 2\sqrt{n-1}(2\delta - c_0)$, then H and A are both bounded on M^n. Therefore, the operator P is bounded, and noticing the assumption $|\nabla H| \in \mathcal{L}^1(M)$, we have

$$|P(\nabla H)| \in \mathcal{L}^1(M). \tag{3.12}$$

Thus, from (3.1), (3.12) and using Lemma 3.2, we obtain that $L(nH) = 0$ on M^n. Then we can reason as in the proof of Theorem 1.1 to conclude that either $|\phi|^2 \equiv 0$ and M^n is totally umbilical, or $S = 2\sqrt{n-1}(2\delta - c_0)$ and M^n has two distinct principle curvatures, one of which is simple. $\square$

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