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Contact surgery and symplectic handlebodies

For Professor Noboru Tanaka
on his 60th birthday

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1. Introduction

The construction and classification of contact manifolds is a basic problem in differential topology. It was shown by Meckert in [5] that the connected sum of two contact manifolds carries a contact structure. (See [8] for applications of Meckert's theorem.) Since the connected sum of two contact manifolds is obtained from their disjoint union (which obviously carries a contact structure) by a simple form of elementary surgery, it is natural to try to extend Meckert's results to more general surgeries. The present paper provides such an extension, while simplifying Meckert's construction as well.

Let X be an orientable contact manifold with contact distribution $\mathscr{D}\subset TX$. $\mathscr{D}$ may be defined by a 1-form α for which $\mathscr{D}=\ker\alpha$ and $d\alpha$ is nondegenerate on $\mathscr{D}$. Such an α is called a *contact form* for the contact structure. The symplectic structure on $\mathscr{D}$ defined by $d\alpha$ is multiplied by a function when α is, and so the vector bundle $\mathscr{D}$ has a natural conformal symplectic structure; in particular, there is a well defined "symplectic orthogonal" operation $\perp'$ on subbundles of $\mathscr{D}$.

A submanifold Y of X is called *isotropic* if all its tangent spaces are contained in $\mathscr{D}$. Since any contact form α vanishes on Y, so does $d\alpha$, so that TY is contained in $(TY)^{\perp'}$. The quotient $(TY)^{\perp'}/TY$ carries a conformal symplectic structure and is called the (conformal) *symplectic normal bundle* of Y. We denote it by $CSN(X, Y)$. The ordinary normal bundle $N(X, Y)=T_YX/TY$ of Y in X is isomorphic to the direct sum of $CSN(X, Y)$, the trivial line bundle $T_YX/\mathscr{D}_Y$, and the quotient $\mathscr{D}_Y/(TY)^{\perp'}$. The last bundle is naturally isomorphic to T^*Y, so if we have a trivial-

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ization of $CSN(X, Y)$ as a conformally symplectic vector bundle and a trivialization of $T_Y X/\mathscr{D}_Y \oplus T^*Y$, we get (up to homotopy) a trivialization of $N(X, Y)$, also called a *framing* of Y in X. In particular, if Y is diffeomorphic (in a given way) to a standard sphere, then $T_Y X/\mathscr{D}_Y \oplus T^*Y$ has a natural trivialization, and so we get a framing induced by each trivialization of $CSN(X, Y)$. Using this framing, we may obtain a new manifold from X by elementary surgery along Y (see [7]).

The main result in this paper is that, if Y is any isotropic sphere in a contact manifold X, with a trivialization of $CSN(X, Y)$, then the manifold X' obtained from X by elementary surgery along Y using the induced framing carries a contact structure. In fact, we will prove more than this. The elementary surgery construction produces a manifold P whose boundary is the disjoint union of X and X'; we will show that this so-called *elementary cobordism* P carries a symplectic structure ω together with a conformally symplectic vector field ξ which is transverse to the boundary. It is this vector field which actually produces the contact structure on X' (and gives back the original contact structure on X).

The basic idea of our construction is the following. An elementary cobordism between manifolds X and X' can be written as the union of a product $X \times I$ (I is an interval), and a *standard handle* which is embedded in $\boldsymbol{R}^{2n}$. When X is a contact manifold, $X \times I$ has a symplectic structure as part of the symplectification of X (See Section 2), while $\boldsymbol{R}^{2n}$ has the standard symplectic structure. Using a normal form for neighborhoods of isotropic submanifolds in contact manifolds (Section 4) we show that these two symplectic structures can be glued together on a neighborhood of the sphere where surgery takes place. In addition, we show that the boundary of the standard handle can be chosen transversal to a conformally symplectic vector field, so that X' inherits a contact structure from the symplectic structure on the cobordism.

This paper owes its existence to remarks by D. McDuff and Y. Eliashberg during the year on Symplectic Geometry and Mechanics at MSRI. It was McDuff who noticed that the "explosion" construction in [12] produced a cobordism between the disjoint union of two copies of a manifold M and their connected sum. Eliashberg showed me the importance of contact manifolds in understanding the structure of noncompact symplectic manifolds (See the discussion at the end of Section 5). I would like to thank both of them for their encouragement. I would also like to thank H. Geiges for pointing out an error in the original manuscript.

2. Liouville vector fields and symplectification

Contact manifolds arise as submanifolds of symplectic manifolds in the following way.

DEFINITION 2.1. *A* Liouville vector field *on a symplectic manifold* (P, ω) *is a vector field* ξ *on* P *for which the Lie derivative* $\mathcal{L}_{\xi}\omega$ *is equal to* $-\omega$.

ξ is a Liouville vector field if and only if $d\alpha=-\omega$, where $\alpha=\xi \lrcorner\, \omega$. It follows (see [10]) that the pullback of α defines a contact structure on any hypersurface transverse to ξ. As the following lemma shows, this contact structure is essentially determined by the Liouville vector field (it lives naturally on the space of trajectories) rather than by the hypersurface.

LEMMA 2.2. *Let* ξ *be a Liouville vector field on the symplectic manifold* (P, ω), *and let* X_0 *and* X_1 *be hypersurfaces which are transverse to* ξ. *Then the local diffeomorphisms from* Y_0 *to* Y_1 *defined by following the integral curves of* ξ *are compatible with the contact structures induced by the form* $\alpha=\xi \lrcorner\, \omega$.

PROOF. The integral curves of ξ leaving X_0 do not all arrive at X_1 at the same time, but we can arrange this to be so (locally), by replacing ξ by $g\xi$, where g is a positive real valued function. This new vector field satisfies

$$\mathcal{L}_{g\xi}\alpha=g\xi \lrcorner\, d\alpha+d(g\xi \lrcorner\, \alpha)=-g\alpha \tag{1}$$

since $\xi \lrcorner\, \alpha=\omega(\xi, \xi)=0$. It follows that the flow of $g\xi$ preserves α up to a conformal factor, and so it is compatible with the contact structures defined by α on hypersurfaces transverse to ξ. □

Any orientable contact manifold can be realized as a hypersurface transverse to a Liouville vector field in its *symplectification* CY. We recall that CY is defined (see for example [1]) to be the submanifold of T^*Y consisting of the values of all contact forms consistent with the orientation. The symplectic structure and standard Liouville field on T^*Y restrict to a symplectic structure and Liouville field ξ on CY. The flow of $-\xi$ defines a free $\boldsymbol{R}$-action on CY which makes it a principal bundle over Y with structure group the real numbers under addition.

Any contact form α for Y is a section of CY whose image is transverse to ξ. The contact structure on Y induced by this section is just the one with which we started. α also determines a trivialization of the bundle CY and hence a diffeomorphism of CY with $Y\times\boldsymbol{R}$. If we denote by t the coordinate on $\boldsymbol{R}$, the image of the symplectic structure and Liouville vector field on $Y\times\boldsymbol{R}$ become $d(e^t\alpha)=e^t(d\alpha+dt\wedge\alpha)$ and $-\frac{d}{dt}$. The section corresponding to α is now defined by the equation $t=0$.

3. The standard handle

In this section, we will construct the "connector" which will eventually be glued to a product $X\times I$ to produce an elementary cobordism.

In the standard symplectic space $\boldsymbol{R}^{2n}$ with canonical symplectic form $\omega=\sum_{i=1}^{2n}dq_i\wedge dp_i$, we define for each $k\in\{0,\dots,n\}$ the Liouville vector field

$$\xi_k=\sum_{i=1}^{n-k}\left(-\frac{1}{2}q_i\frac{\partial}{\partial q_i}-\frac{1}{2}p_i\frac{\partial}{\partial p_i}\right)+\sum_{i=n-k+1}^{n}\left(-2q_i\frac{\partial}{\partial q_i}+p_i\frac{\partial}{\partial p_i}\right) \tag{2}$$

which is the negative gradient with respect to the standard euclidean metric of the Morse function

$$f_k=\sum_{i=1}^{n-k}\left(\frac{1}{4}q_i^2+\frac{1}{4}p_i^2\right)+\sum_{i=n-k+1}^{n}\left(q_i^2-\frac{1}{2}p_i^2\right) \tag{3}$$

To see that this is indeed a Liouville vector field, we observe that the contraction

$$\alpha_k=\xi_k\lrcorner\,\omega=\sum_{i=1}^{n-k}\left(-\frac{1}{2}q_idp_i+\frac{1}{2}p_idq_i\right)+\sum_{i=n-k+1}^{n}(-2q_idp_i-p_idq_i) \tag{4}$$

satisfies $d\alpha_k=-\omega$.

In what follows, we will systematically use equations contained in braces to denote the set of solutions to those equations. For example, the unstable manifold E_-^k is $\{q_1=\cdots=q_n=p_1=\cdots=p_{n-k}=0\}$. Since the form α_k pulls back to zero on E_-^k, the *descending sphere* $S^{k-1}=E_-^k\cap\{f_k=-1\}$ is an isotropic submanifold in the contact manifold $X_{-1}=\{f_k=-1\}$. Similarly, in the stable manifold $E_+^{2n-k}=\{p_{n-k+1}=\cdots=p_n=0\}$, the *ascending sphere* $S_+^{2n-k-1}=E_+^{2n-k}\cap\{f_k=1\}$ is a submanifold in the contact manifold $X_+=\{f_k=1\}$.

A standard handle in $\boldsymbol{R}^{2n}$ is a region bounded by a neighborhood of the descending sphere in X_- together with a connecting manifold Σ diffeomorphic to $S^{2n-k-1}\times D^k$. As is suggested in Figure 1 and may be verified using Lemma 3.1. below, this handle can be chosen so that it is transverse to the Liouville vector field ξ_k and so that its intersection with

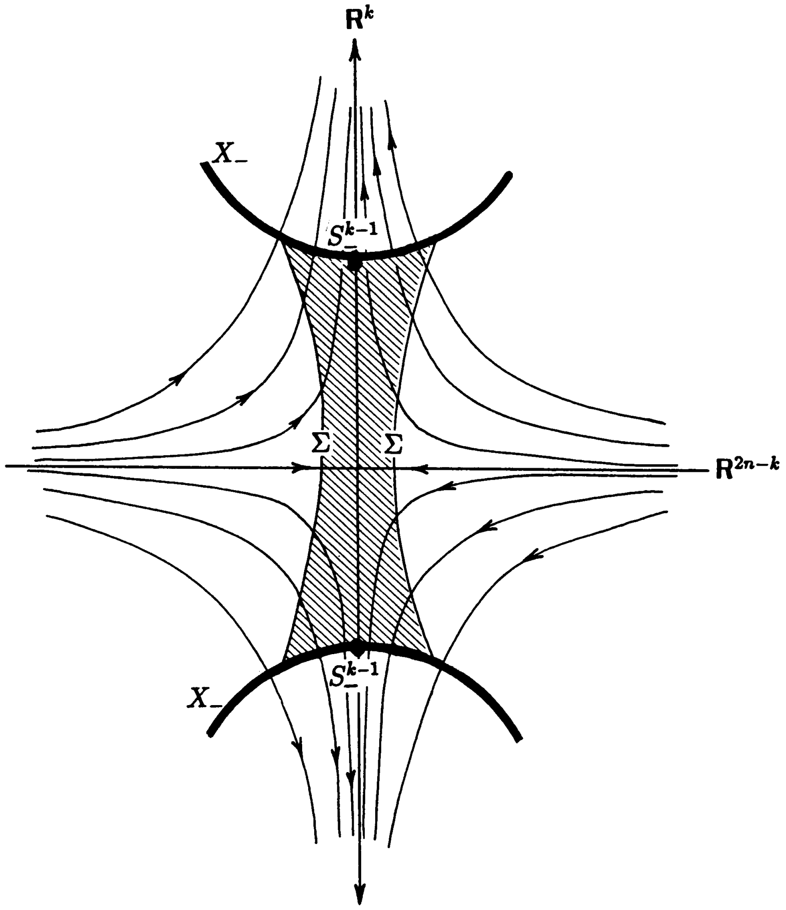

FIGURE 1. The boundary of the standard handle (shaded) is transverse to the Liouville vector field.

X_- is contained in an arbitrarily small neighborhood of the descending sphere. As a result, the union of this handle with $f_k \leq -1$ is a symplectic manifold whose boundary is everywhere transverse to ξ_k and hence a contact manifold obtained from X_- by surgery in an arbitrarily small neighborhood of the descending sphere.

LEMMA 3.1. *On $\boldsymbol{R}^{2n}$, denote the coordinates $(q_1, \ldots, q_n, p_1, \ldots, p_{n-k})$ by $(x_1, \ldots, x_{2n-k})$ and $(p_{n-k+1}, \ldots, p_n)$ by $(y_1, \ldots, y_k)$. Let the hypersurface Σ in $\boldsymbol{R}^{2n}$ be defined by an equation of the form $F(\sum A_i x_i^2, \sum B_j y_j^2)=0$, where the coefficients A_i and B_j are positive. Suppose further that, whenever $F(x, y)=0$, the partial derivatives of F do not have the same sign, that $\frac{\partial F}{\partial x}$ is not zero when $y=0$, that $\frac{\partial F}{\partial y}$ is not zero when $x=0$, and finally that $F(0, 0)\neq 0$. Then Σ is transverse to the Liouville vector field ξ_k.*

PROOF. ξ_k has the form $-\sum a_i x_i \frac{\partial}{\partial x_i} + \sum b_j y_j \frac{\partial}{\partial y_j}$ with positive coefficients a_i and b_j. The derivative $\xi_k \cdot F(\sum A_i x_i^2, \sum B_j y_j^2)$ equals twice $\left(-\frac{\partial F}{\partial x}\sum a_i A_i x_i^2 + \frac{\partial F}{\partial y}\sum b_j B_j y_j^2\right)$. The assumptions on F insure that this expression is never 0 on Σ. □

4. Neighborhoods of isotropic submanifolds

In order to glue the standard handle onto a given contact manifold, we find in this section a normal form for a neighborhood of Y in a quintuple (P, ω, ξ, X, Y), where (P, ω) is a symplectic manifold, ξ is a Liouville vector field, X is a hypersurface transverse to ξ (hence a contact manifold), and Y is an isotropic submanifold of X. For convenience, we shall call such a quintuple an *isotropic setup*.

Given an isotropic setup, $\alpha = \xi \lrcorner \omega$ is a contact form on X, and so the symplectic normal bundle $CSN(X, Y)$ defined in Section 1 has a (not just conformally) symplectic structure. We call this symplectic vector bundle the *symplectic subnormal bundle* of the isotropic setup.

REMARK 4.1. Y is also an isotropic submanifold of the symplectic manifold P. As such, it has an ordinary symplectic normal bundle $SN(P, Y) = TY^{\perp}/TY$, which is isomorphic to the direct sum of the symplectic subnormal bundle and the trivial 2-dimensional symplectic vector bundle with canonical basis (ξ, η), where η is the *Reeb vector field* on X defined by the conditions $\eta \lrcorner d\alpha = 0$ and $\eta \lrcorner \alpha = 1$.

Any isomorphism between two isotropic setups $(P_0, \omega_0, \xi_0, X_0, Y_0)$ and $(P_1, \omega_1, \xi_1, X_1, Y_1)$ obviously induces a diffeomorphism from Y_0 to Y_1 which is covered by an isomorphism of their symplectic subnormal bundles. The (partial) converse to this fact is given by the following proposition, which is close to but not contained in Theorem 1.3.B in Chapter 4 of [2].

PROPOSITION 4.2. *Let $(P_0, \omega_0, \xi_0, X_0, Y_0)$ and $(P_1, \omega_1, \xi_1, X_1, Y_1)$ be isotropic setups. Given a diffeomorphism from Y_0 to Y_1 covered by an isomorphism between their symplectic subnormal bundles, there exist neighborhoods U_j of Y_j in P_j and an isomorphism of isotropic setups*

$$\phi : (U_0, \omega_0, \xi_0, X_0 \cap U_0, Y_0) \to (U_1, \omega_1, \xi_1, Y_1 \cap U_1, Y_1)$$

which restricts to the given mappings on Y_0.

PROOF. For $i = 0, 1$, we choose a neighborhood V_i of Y_i in X_i and a hypersurface Z_i in V_i such that Z_i contains Y_i and is tangent to the contact distribution along Y_i. By taking V_i small enough, we can arrange that Z_i is symplectic as a submanifold of P_i and that Z_i intersects each integral curve of the Reeb vector field η_i exactly once.

If we consider Y_i as an isotropic submanifold of Z_i, its symplectic normal bundle $SN(Z_i, Y_i)$ is naturally isomorphic to the symplectic subnormal bundle $SN(P_i, X_i, Y_i)$, so by the semilocal equivalence theorem for isotropic submanifolds in symplectic manifolds [9], we can find (shrinking Z_i and V_i, if necessary) a symplectic diffeomorphism ψ from Z_0 to Z_1 which induces the given diffeomorphism and bundle mapping on Y_0. We then extend ψ to a diffeomorphism (again called ψ) from V_0 to V_1 by requiring that it map integral curves of η_0 to those of η_1.

Let β_0 be the contact form on V_0 which is the pullback of $\alpha_0=\xi_0 \lrcorner\, \omega_0$, and let $\beta_1=\psi^*\alpha_1$ be the pullback of the contact form on V_1. By the construction of ψ, we have $d\beta_0=d\beta_1$ on V_0, and $\beta_0=\beta_1$ on Y_0. We will next find a diffeomorphism χ between neighborhoods of Y_0 in V_0, tangent to the identity along Y_0, such that $\chi^*\beta_1=\beta_0$.

Define β_t to be $\beta_0+t(\beta_1-\beta_0)$ for $t\in[0,1]$. Shrinking V_0 if necessary, we can assume that each β_t is a contact form. To use the deformation method as in [9], it will suffice to find for each t a vector field v_t vanishing to second order along Y_0 such that $d(v_t \lrcorner\, \beta_t)+v_t \lrcorner\, d\beta_t=\beta_0-\beta_1$. By our conditions on β_0 and β_1, we can write $\beta_0-\beta_1$ as df, where f is a function which vanishes to second order along Y_0. Now choose v_t to be f times the Reeb vector field of β_t, and integrate the time dependent vector field v_t to obtain χ.

Composing χ with ψ and shrinking neighborhoods again, we get a diffeomorphism ϕ from V_0 to V_1 which pulls back the contact form α_1 to α_0. Finally, we extend ϕ to a diffeomorphism between neighborhoods U_i of Y_i in P_i by requiring it to map integral curves of the Liouville vector field ξ_0 to those of ξ_1. It is now easy to check for this extended ϕ, $\phi^*\alpha_1$ agrees with α_0 along X_0. Since both of the 1-forms α_0 and $\phi^*\alpha_1$ satisfy the Lie derivative equation $\mathscr{L}_{\xi_0}\alpha=-\alpha$, they must be equal, so ϕ is the required symplectomorphism.

For completeness, although we will not use this result, we give a "canonical model" for an isotropic setup with given symplectic subnormal bundle.

PROPOSITION 4.3. *There is a functorial construction which associates to each manifold Y and symplectic vector bundle E over Y an isotropic setup whose symplectic subnormal bundle is naturally isomorphic to E.*

PROOF. We recall the construction in [11] which associates to each symplectic vector bundle E over a manifold Y a symplectic manifold $\mathscr{S}E$ with an isotropic embedding of Y and an isomorphism of E with its symplectic normal bundle. Let B be the principal $Sp(2m)$ bundle of which E is an associated $\boldsymbol{R}^{2m}$ bundle, with the usual right action of $Sp(2m)$ on B converted into a left action by inversion in the group. The symplectic manifold $T^*B\times\boldsymbol{R}^{2m}$ then has a left action of $Sp(2m)$. Symplectic reduction at $0\in\mathfrak{sp}(2m)^*$ produces the symplectic manifold $\mathscr{S}E$. Y is identified with the image of the zero section in T^*B times the zero element of $\boldsymbol{R}^{2m}$.

We observe now that $T^*B\times\boldsymbol{R}^{2n}$ carries an $Sp(2m)$-invariant Liouville vector field ξ' generating the flow which, for each $t>0$, multiplies covectors in T^*B by e^{-t} and elements of $\boldsymbol{R}^{2m}$ by $e^{-\sqrt{t}}$. The momentum mapping for the $Sp(2m)$ action on $T^*B\times\boldsymbol{R}^{2n}$ is linear on fibres of T^*B and quadratic on $\boldsymbol{R}^{2m}$, so its zero level set is invariant under ξ', and hence ξ' projects to a Liouville vector field on $\mathscr{S}E$.

Finally, we obtain the isotropic setup (P, ω, ξ, X, Y) by letting (P, ω) be $\mathscr{S}E\times\boldsymbol{R}^2$ with its product symplectic structure, ξ the product of ξ' with the radial Liouville vector field on $\boldsymbol{R}^2$, X the product of $\mathscr{S}E$ with the unit circle in $\boldsymbol{R}^{2n}$, and Y the product of the copy of Y in $\mathscr{S}E$ with the point $(1, 0)$ in $\boldsymbol{R}^2$. □

5. Elementary cobordisms

We are ready to prove our main theorem.

THEOREM 5.1. *Let Y be an isotropic sphere in the contact manifold X with a trivialization of $CSN(X, Y)$. Let X' be the manifold obtained from X by elementary surgery along Y. Then the elementary cobordism P from X to X' obtained by attaching a standard handle to $X\times I$ along a neighborhood of Y carries a symplectic structure and a Liouville vector field which is transverse to X and X'. The contact structure induced on X is the given one, while that on X' differs from that on X only on the spheres where the surgery takes place.*

PROOF. We will use Proposition 4.2 to glue a standard handle to the product $X\times I$, where I is the interval $[-1, 0]$. For the first isotropic setup to which the theorem will be applied, we let P_0 be $X\times\boldsymbol{R}$ with the symplectic structure $\omega_0=d(e^t\alpha)$ obtained from the symplectification of X, using a particular contact form α. The Liouville vector field ξ_0 is just

$-\frac{d}{dt}$, X_0 is $X\times\{0\}$, and Y_0 is $Y\times\{0\}$. The symplectic subnormal bundle of this isotropic setup is trivialized by the assumption in the theorem.

Let the dimension of the sphere Y be $k-1$. For the second isotropic setup, we use the data associated with the standard handle as defined in Section 3. P_1 is $\boldsymbol{R}^{2n}$, ω_1 is the standard symplectic structure. ξ_1 the Liouville vector field defined in Equation 2 and denoted there by ξ_k. The transverse hypersurface X_1 is the level manifold X_-, and the isotropic submanifold Y_1 is the descending sphere S_-^{k-1}. Its symplectic subnormal bundle is trivialized by the vector fields $\frac{\partial}{\partial q_i}$ and $\frac{\partial}{\partial p_i}$ for $i=1,\dots,n-k$.

Applying Proposition 4.2 now gives us an isomorphism (see Figure 2) of isotropic setups between a neighborhood U_0 of Y_0 in P_0 and a neighborhood U_1 of Y_1 in P_1. Using this isomorphism, as shown in Figure 3, we can make a smooth manifold P of the union of $X\times I$ and the standard handle in $\boldsymbol{R}^{2n}$. P inherits from its pieces a symplectic structure ω and a Liouville vector field ξ which is transverse to each of the boundary components $X\times\{-1\}$ and X'. The first of these components is contact-diffeomorphic to X. The second is diffeomorphic to the result of elementary surgery on X and inherits a contact structure from ω and ξ. A contact diffeomorphism from most of X to most of X' (with just the descending and ascending spheres omitted) is given by flowing along ξ, thanks to Lemma 2.2. □

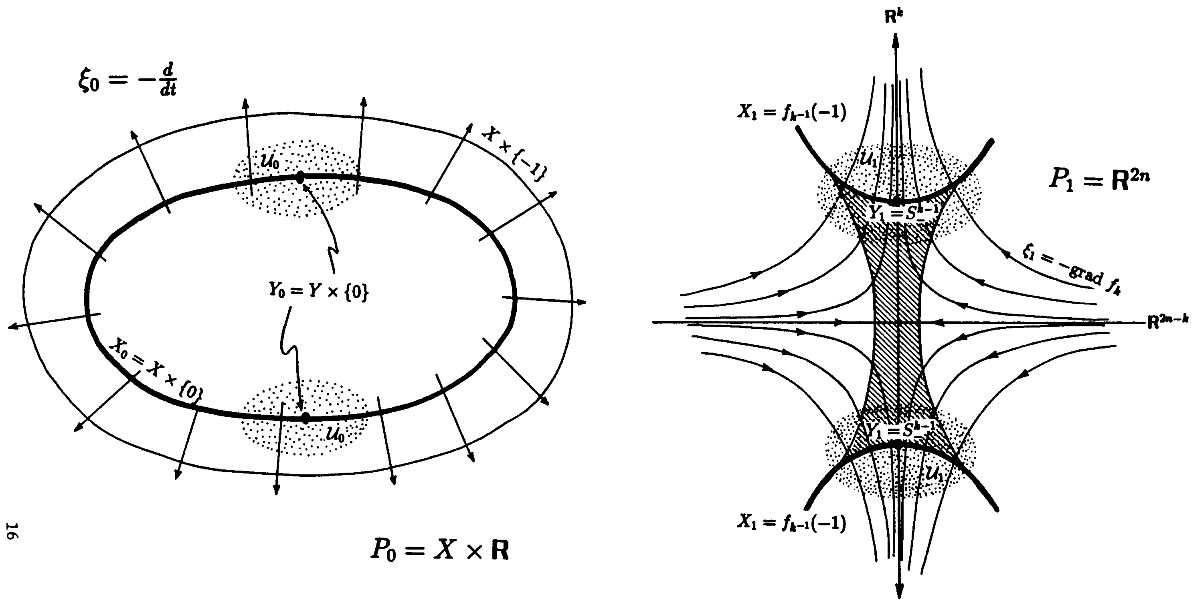

FIGURE 2. Isomorphic neighborhoods of isotropic spheres in $X\times I$ and $\boldsymbol{R}^{2n}$.

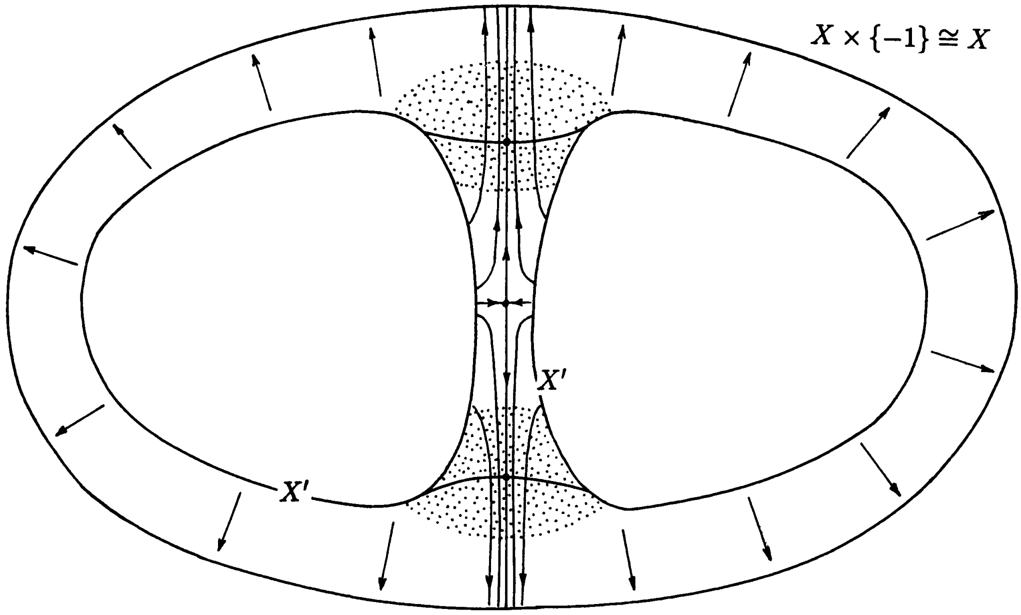

FIGURE 3. The elementary cobordism obtained by gluing the standard handle to $X \times [-1, 0]$.

We conclude this paper with a discussion of its relation to recent work of Y. Eliashberg. It is convenient to make the following definition.

DEFINITION 5.2. *A* Liouville pair *on a symplectic manifold consists of a Liouville vector field ξ and a function f for which the critical point set of f coincides with the zero set of ξ, with $\xi \cdot f < 0$ except on this set. The Liouville pair is* nondegenerate *if ξ is hyperbolic at each of its zeros and f is a Morse function.*

When (ξ, f) is a nondegenerate Liouville pair on P, the expanding subspace for ξ at each of its zeros is isotropic, so the index of each critical point of f can be at most $\frac{1}{2}\dim P$. Following Eliashberg and Gromov [4], we define a *Stein symplectic manifold* to be a symplectic manifold admitting a nondegenerate Liouville pair in which the vector field is complete and the function is proper and bounded from below. It follows from basic Morse theory [6] that a Stein symplectic manifold P satisfies the same topological restriction as the Stein manifolds of complex analysis: it has the homotopy type of a simplicial complex of dimension at most $\frac{1}{2}$ $\dim P$.

Conversely, one can attempt to build up Stein symplectic manifolds by gluing together elementary cobordisms carrying Liouville pairs as constructed in Theorem 5.1. In fact, Eliashberg and Gromov [3][4] have shown that the homotopy condition above, together with the existence of an almost complex structure, is sufficient for the existence of either kind of Stein structure on a manifold P when when $\dim P > 4$. Our work in this paper shmplifies part of Eliashberg's construction in the symplectic case.

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