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SHIFTED JACK POLYNOMIALS AND THEIR APPLICATIONS

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The present talk is based on the joint papers with Andrei Okounkov [OO3, OO4].

1. Jack polynomials ([M,St]). We will use the parameter

$$\theta = 1/\alpha$$

inverse to the standard parameter α for Jack polynomials. Jack symmetric polynomials $P_\lambda(x_1,\dots,x_n;\theta)$ are eigenfunctions of Sekiguchi differential operators

$$D(u;\theta) = V(x)^{-1}\det\left[x_i^{n-j}\left(x_i\frac{\partial}{\partial x_i} + (n-j)\theta + u\right)\right]_{1\le i,j\le n},$$

$$D(u;\theta)\,P_\lambda(x;\theta) = \left(\prod_i(\lambda_i + (n-i)\theta + u)\right)P_\lambda(x;\theta)\,, \tag{1.1}$$

where $V(x) = \prod_{i<j}(x_i - x_j)$ is the Vandermonde determinant and u is a parameter. The operators $\{D(u;\theta), u\in\mathbb{C}\}$ generate a commutative algebra: denote it by $\mathfrak{D}(n;\theta)$.

We normalize $P_\lambda(x_1,\dots,x_n;\theta)$ so that

$$P_\lambda(x_1,\dots,x_n;\theta) = x_1^{\lambda_1}\dots x_n^{\lambda_n} + \dots\,,$$

where dots stand for lower monomials in lexicographic order. Then one has

$$P_{(\lambda_1+1,\dots,\lambda_n+1)}(x;\theta) = \left(\prod x_i\right)P_\lambda(x;\theta)\,. \tag{1.2}$$

Using (1.2), one can define *Laurent* Jack polynomials for any n-tuples of integers $\lambda_1\ge\dots\ge\lambda_n$ (called *signatures*), and still (1.1), and hence the binomial theorem below, will hold.

Note that the eigenvalues in (1.1) are symmetric in variables $\lambda_i - \theta i$, $i=1,\dots,n$.

2. Shifted Jack polynomials ([S1,OO,Ok1,KS,Ok3]). Denote by $\Lambda^\theta(n)$ the algebra of polynomials $f(x_1,\dots,x_n)$ that are symmetric in variables $x_i - \theta i$. We call such polynomials *shifted symmetric*. By (1.1) one has the Harish-Chandra isomorphism

$$\mathfrak{D}(n;\theta)\to\Lambda^\theta(n) \tag{2.1}$$

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which takes an operator $D \in \mathfrak{D}(n;\theta)$ to the polynomial $d \in \Lambda^\theta(n)$ such that

$$D\,P_\lambda(x;\theta) = d(\lambda)\,P_\lambda(x;\theta)\,.$$

If D is of order k then $\deg d = k$.

Let μ be a partition (also viewed as a diagram). Recall that for a square $s = (i,j) \in \mu$ the numbers

$$\begin{aligned} a(s) &= \mu_i - j, & a'(s) &= j-1, \\ l(s) &= \mu'_j - i, & l'(s) &= i-1, \end{aligned}$$

are called arm-length, arm-colength, leg-length, and leg-colength respectively. Set

$$H(\mu) = \prod_{s\in\mu}(a(s) + \theta\, l(s) + 1)\,.$$

According to a general result of S. Sahi [S1] there exists the unique polynomial $P^*_\mu(x;\theta)$ in $\Lambda^\theta(n)$ such that $\deg P^*_\mu \le |\mu|$ and

$$P^*_\mu(\lambda;\theta) = \begin{cases} H(\mu), & \lambda = \mu\,, \\ 0, & |\mu| \le |\lambda|,\ \mu \ne \lambda \end{cases}$$

Here we assume μ and λ have length $\le n$. It is clear that the uniqueness of $P^*_\mu(x,\theta)$ follows easily from its existence.

The polynomials $P^*_\mu(x;1)$ were studied in [OO] and [Ok1]. They are closely related to Schur polynomials and are called *shifted Schur functions.* In particular, the Schur function s_μ is the highest degree term of $P^*_\mu(x;1)$. Shifted Schur functions have a very rich combinatorics and numerous applications (such as Capelli-type identities or asymptotic character theory, see [OO,Ok1,N,Ok2] and references therein).

For general θ, F. Knop and S. Sahi proved (see [KS], Theorem 5.2) that

$$P^*_\mu(\lambda;\theta) = 0\,, \quad \text{unless} \quad \mu \subset \lambda\,, \tag{2.2}$$

and that again ([KS], Corollary 4.7)

$$P^*_\mu(x;\theta) = P_\mu(x;\theta) + \quad \textit{lower degree terms}\,. \tag{2.3}$$

Their proof was based on difference equations for the polynomial $P^*_\mu(x;\theta)$. For $\theta = 1$ these properties follow immediately from explicit formulas for $P^*_\mu(x;1)$.

Explicit formulas for the polynomials $P^*_\mu(x;\theta)$, which generalize explicit formulas for shifted Schur functions, were found in [Ok3] (see, for example, (2.4) below). In particular, they provide an effective proof of the existence of $P^*_\mu(x;\theta)$ and a different proof of (2.2) and (2.3).

We call these polynomials *shifted Jack polynomials.* They are a degenerate case of shifted Macdonald polynomials [Kn,S2,Ok3].

It easily follows from the definition that $P^*_\mu(x;\theta)$ are stable, that is

$$P^*_\mu(x_1,\dots,x_n,0;\theta) = P^*_\mu(x_1,\dots,x_n;\theta)$$

provided μ has at most n parts. Hence, one can consider shifted Jack polynomials in infinitely many variables.

The following *combinatorial formula* for shifted Jack polynomials was proved in [Ok3]. Let us call a tableau T on μ a *reverse tableau* if its entries strictly decrease down the columns and weakly decrease in the rows. By $T(s)$ denote the entry in the square $s \in \mu$. We have

$$P^*_\mu(x;\theta) = \sum_T \psi_T(\theta) \prod_{s\in\mu} \left(x_{T(s)} - a'(s) + \theta\, l'(s)\right) , \tag{2.4}$$

where the sum is over all reverse tableaux on μ with entries in $\{1,2,\dots\}$ and $\psi_T(\theta)$ is the same θ-weight of a tableau that enters the combinatorial formula for ordinary Jack polynomials (see [St] or [M],(VI.10.12))

$$P_\mu(x;\theta) = \sum_T \psi_T(\theta) \prod_{s\in\mu} x_{T(s)} \,. \tag{2.5}$$

The coefficients $\psi_T(\theta)$ are rational functions of θ.

3. Binomial formula. Given a partition μ and a number t, set

$$\begin{aligned} H'(\mu) &= \prod_{s\in\mu} (a(s) + \theta\, l(s) + \theta) \,, \\ (t)_\mu &= \prod_{s\in\mu} (t + a'(s) - \theta\, l'(s)) \,. \end{aligned}$$

If $\mu = (m)$ then $(t)_\mu$ is the standard shifted factorial. We have (see [M], VI.10.20)

$$P_\mu(1,\dots,1;\theta) = (n\theta)_\mu / H'(\mu) \,. \tag{3.1}$$

Set (see [M], VI.10.16)

$$Q_\mu(x;\theta) = \frac{H'(\mu)}{H(\mu)} P_\mu(x;\theta) \,.$$

Theorem.

$$\frac{P_\lambda(1+x_1,\dots,1+x_n;\theta)}{P_\lambda(1,\dots,1;\theta)} = \sum_\mu \frac{P^*_\mu(\lambda_1,\dots,\lambda_n;\theta)\, Q_\mu(x_1,\dots,x_n;\theta)}{(n\theta)_\mu} \,. \tag{3.2}$$

This result is proved in [OO3]. Note that by virtue of (2.2) the summation in the binomial formula (3.2) is only over such μ that $\mu \subset \lambda$.

Remark. The coefficients $\binom{\lambda}{\mu}_\theta$ in the expansion

$$\frac{P_\lambda(1+x_1,\dots,1+x_n;\theta)}{P_\lambda(1,\dots,1;\theta)}=\sum_\mu \binom{\lambda}{\mu}_\theta \frac{P_\mu(x_1,\dots,x_n;\theta)}{P_\mu(1,\dots,1;\theta)}.$$

are called the *generalized binomial coefficients.* They were studied by C. Bingham [Bi] in the special case $\theta=1/2$, using group-theoretical methods, and by M. Lassalle [La] for general θ. Then a proof of the main results announced in [La] was proposed by J. Kaneko (see [K], section 5). The theorem says that this binomial coefficient

$$\binom{\lambda}{\mu}_\theta=\frac{P^*_\mu(\lambda;\theta)}{H(\mu)} \tag{3.3}$$

is a shifted symmetric polynomial in λ which by (2.4) and (2.5) is just as complex as the Jack polynomial $P_\mu(x;\theta)$ itself. The main results of Lassalle can be easily deduced from (3.3). For example, it is clear that (3.3) does not depend on n and vanishes unless $\mu\subset\lambda$. The recurrence relation (see [La], théorèmes 2, 3, 5 and corollaire in section 6) is equivalent to the formula (5.2) below.

Note also that the coefficients (3.3) admit a simple formula when $\theta=1$ (see A. Lascoux [Lasc] and Example 10 in [M], section I.3) or when $\lambda_1=\cdots=\lambda_n$ (see J. Faraut and A. Korányi [FK], Prop. XII.1.3 (ii)).

Binomial formulas for characters of classical groups are discussed in [OO2]. A. Okounkov found a binomial formula for Macdonald polynomials (both shifted and ordinary) and also for Koornwinder polynomials, see [Ok4-5].

4. Bessel functions ([D,Op,J]). For a real vector $l=(l_1,\dots,l_n)$ denote by

$$[l]=([l_1],\dots,[l_n])$$

its integral part. Suppose that $l_1\ge\cdots\ge l_n$. By definition, put

$$F(l,x;\theta)=\lim_{\kappa\to\infty}\frac{P_{[\kappa l]}(1+x_1/\kappa,\dots,1+x_n/\kappa;\theta)}{P_{[\kappa l]}(1,\dots,1;\theta)}. \tag{4.1}$$

From (3.2) and (2.3) we get (see [OO3])

Proposition.

$$F(l,x;\theta)=\sum_\mu \frac{P_\mu(l_1,\dots,l_n;\theta)\,Q_\mu(x_1,\dots,x_n;\theta)}{(n\theta)_\mu}. \tag{4.2}$$

We call $F(l,x;\theta)$ the *Bessel functions.* They are in the same relation to Jack polynomials as ordinary Bessel functions to Jacobi polynomials. They are eigenfunctions of the corresponding degeneration of Sekiguchi operators. The formula (4.2) makes obvious the following known symmetry

$$F(l,x;\theta)=F(x,l;\theta).$$

Let $H(n,\mathbb{R})$ denote the space of real symmetric matrices of order n, let $X, Y \in H(n,\mathbb{R})$, and let $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ be the spectra of X and Y. The compact orthogonal group $O(n)$ acts on the space $H(n,\mathbb{R})$ by conjugations. One can prove that

$$F(y, x; 1/2) = \int_{O(n)} \exp(\operatorname{tr}(YuXu^{-1}))du, \tag{4.3}$$

where du is the normalized Haar measure on $O(n)$.

(Idea of proof: the normalized polynomials

$$P_\lambda(z_1, \ldots, z_n; 1/2)/P_\lambda(1, \ldots, 1; 1/2)$$

are the spherical functions on the compact symmetric space $U(n)/O(n)$ while the integrals (4.3) are essentially the spherical functions on the associated Euclidean type symmetric space $O(n) \ltimes H(n,\mathbb{R})$. It is well–known (see [DR]) that the spherical functions of Euclidean type can be obtained from the spherical functions of compact type by the limit transition (4.1).)

A similar interpretation of the functions $F(y, x; \theta)$ also exists for $\theta = 1$ and $\theta = 2$. Then one has to consider orbits in the spaces $H(n,\mathbb{C})$ and $H(n,\mathbb{H})$ of complex and quaternionic Hermitian matrices, respectively. An application of the expansion (4.2) for $\theta = 1$ is given in [OV]; similar results also hold for $\theta = 1/2$ and $\theta = 2$.

5. Formula for θ-dimension of a skew diagram λ/μ. Define the θ-dimension $\theta\text{-}\dim \lambda/\mu$ of a skew diagram λ/μ as the following coefficient

$$\Big(\sum x_i\Big)^k P_\mu(x;\theta) = \sum_{|\lambda|=|\mu|+k} \theta\text{-}\dim \lambda/\mu \; P_\lambda(x;\theta)\,, \quad k = 1, 2, \ldots . \tag{5.1}$$

If $\theta = 1$ then $\theta\text{-}\dim \lambda/\mu$ equals the number of the standard tableaux on λ/μ; for general θ each tableau T is counted with a certain weight $\psi'_T(\theta)$ given in [M], section VI.6. In particular, if $\mu = \emptyset$ then

$$\theta\text{-}\dim \lambda = |\lambda|!/H(\lambda)\,.$$

We have

Proposition.

$$\frac{\theta\text{-}\dim \lambda/\mu}{\theta\text{-}\dim \lambda} = \frac{P^*_\mu(\lambda;\theta)}{|\lambda|(|\lambda|-1)\cdots(|\lambda|-|\mu|+1)}\,. \tag{5.2}$$

This proposition can be deduced from (3.3) and [La,K] (see the remark above), but it easier to give a direct proof, which uses the very same argument as in [OO], section 9 (see [OO3]).

6. Integral representation of Jack polynomials. Write $\nu \prec \lambda$ if

$$\lambda_1 \geq \nu_1 \geq \lambda_2 \geq \nu_2 \geq \cdots \geq \nu_{n-1} \geq \lambda_n .$$

For any number r set $(t)_r = \Gamma(t+r)/\Gamma(t)$. We have (see [M], VI.7.13′)

$$P_\lambda(x_1,\dots,x_n;\theta) = \sum_{\nu\prec\lambda} \psi_{\lambda/\nu} P_\nu(x_1,\dots,x_{n-1}) x_n^{|\lambda/\nu|}, \tag{6.1}$$

where

$$\psi_{\lambda/\nu} = \prod_{i\leq j\leq n-1} \frac{(\nu_i - \lambda_{j+1} + \theta(j-i)+1)_{\theta-1}(\lambda_i - \nu_j + \theta(j-i)+1)_{\theta-1}}{(\lambda_i - \lambda_{j+1} + \theta(j-i)+1)_{\theta-1}(\nu_i - \nu_j + \theta(j-i)+1)_{\theta-1}}. \tag{6.2}$$

Suppose μ has less than n parts. The formulas (3.2) and (6.1) together imply

$$P_\mu^*(\lambda) = \frac{(n\theta)_\mu}{((n-1)\theta)_\mu} \sum_{\nu\prec\lambda} \psi_{\lambda/\nu} \frac{P_\nu(1,\dots,1)}{P_\lambda(1,\dots,1)} P_\mu^*(\nu) \tag{6.3}$$

where 1 is repeated n times in the denominator and $(n-1)$ times in the numerator. Note that after substitution of (6.2) and of (3.1) rewritten as

$$P_\lambda(1,\dots,1) = \prod_{i<j\leq n} (\lambda_i - \lambda_j + \theta(j-i))_\theta \prod_{i\leq n} \Gamma(\theta)/\Gamma(\theta i) \tag{6.4}$$

the formula (6.3) becomes the formula (7.16) from [Ok3]. We replace λ and μ in (6.3) by their multiples $\kappa\lambda$ and $\kappa\mu$ and let $\kappa \to \infty$. By (2.3)

$$P_\mu^*(\kappa\lambda)/\kappa^{|\mu|} \to P_\mu(\lambda), \quad \kappa \to \infty .$$

Introduce the following product of beta-functions

$$C(\mu,n) = \prod_{i\leq n-1} B(\mu_i + (n-i)\theta, \theta).$$

Set

$$\Pi(\lambda,\nu;\theta) = \prod_{i\leq j} (\lambda_i - \nu_j)^{\theta-1} \prod_{i>j} (\nu_j - \lambda_i)^{\theta-1}.$$

Using the well-known relation $(t)_\theta / t^\theta \to 1$, $t \to \infty$, one obtains from (6.3) the following

Proposition. *If μ has $< n$ parts then*

$$P_\mu(\lambda) = \frac{1}{C(\mu,n)} \frac{1}{V(\lambda)^{2\theta-1}} \int_{\lambda_2}^{\lambda_1} d\nu_1 \dots \int_{\lambda_n}^{\lambda_{n-1}} d\nu_{n-1}\, P_\mu(\nu)\, V(\nu)\, \Pi(\lambda,\nu;\theta).$$

Here $\lambda_1 \geq \cdots \geq \lambda_n$ are real (actually, the ordering is not essential since P_μ is symmetric). If $\theta = 1,2,\dots$ then the integrand is holomorphic and λ can be arbitrary complex. Iteration of this proposition together with (1.2) gives an integral representation for all Jack polynomials.

This proposition was first obtained by the author (unpublished) for special values $\theta = 1/2, 1, 2$ by using the group theoretic interpretation of the corresponding Jack polynomials. Then A. Okounkov [Ok3] found a general method which works for any θ and even for Macdonald polynomials. The proof given in [OO3] differs from that from [Ok3].

7. Application to asymptotics of Jack polynomials. The binomial theorem results in a θ-analog of the Vershik-Kerov theorem [VK2] about characters of $U(\infty)$.

Consider normalized (Laurent) Jack polynomials

$$\frac{P_\lambda(z_1,\dots,z_n;\theta)}{P_\lambda(1,\dots,1;\theta)} \tag{7.1}$$

living on the n-dimensional torus

$$\mathbb{T}^n := \{z=(z_1,\dots,z_n) \mid |z_1|=\dots=|z_n|=1\}$$

We embed $\mathbb{T}^n$ into $\mathbb{T}^{n+1}$ via $z\mapsto(z,1)$ and define the infinite-dimensional torus $\mathbb{T}^\infty$ as the inductive limit of the tores $\mathbb{T}^n$.

Problem. Describe all functions Φ on $\mathbb{T}^\infty$ which can be obtained as limits (uniformly on each finite-dimensional torus) of normalized Jack polynomials (7.1), where $n\to\infty$ and λ, a signature of length n, varies with n.

For 3 special values $\theta=1/2,1,2$, the limit functions Φ are exactly indecompozable spherical functions on infinite-dimensional symmetric spaces of compact type G/K, where G is one of the groups $U(\infty)$, $U(\infty)\times U(\infty)$, $U(2\infty)$, and K is its subgroup isomorphic to $O(\infty)$, diagonal $U(\infty)$, $Sp(\infty)$, respectively.

A solution to the problem was given in [OO4]. It turns out that the limit functions Φ are of the form

$$\Phi(z_1,z_2,\dots)=\varphi(z_1)\varphi(z_2)\dots,$$

where $\varphi(z)$ is a real-analytic function on the circle $\mathbb{T}^1$, $\varphi(1)=1$, depending on 4 series of nonnegative real parameters $\alpha_i^\pm$, $\beta_i^\pm$, and two nonnegative real parameters $\gamma^\pm$. Specifically,

$$\varphi(z)=\varphi_+(z)\varphi_-(z),$$

where $\varphi_+(z)$ is holomorphic for $|z|<1$, $\varphi_-(z)$ is holomorphic for $|z|>1$, and

$$\varphi_\pm(z)=e^{\gamma^\pm(z^{\pm1}-1)}\prod_{i=1}^\infty(1+\beta_i^\pm(z^{\pm1}-1))\left(1-\frac{\alpha_i^\pm(z^{\pm1}-1)}{\theta}\right)^{-\theta}.$$

8. Application to θ-version of characters of the infinite symmetric group. There exists a similar result related to the infinite symmetric group $S(\infty)$. It concerns a θ-version of Thoma–Vershik–Kerov's theorem on the characters of $S(\infty)$ (see [VK1]). Such a result was obtained in [KOO] by making use of formula (5.2).

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