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Yangian of the queer Lie superalgebra'

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Abstract. Consider the complex matrix Lie superalgebra $\mathfrak{gl}_{n|n}$ with the standard generators e_{ij}; $i,j \in \mathbf{Z}_{2n}$. Define an involutory automorphism π of $\mathfrak{gl}_{n|n}$ by $\pi(e_{ij}) = e_{i+n,j+n}$. Dissimilarly to the even case, the twisted polynomial current Lie superalgebra

$$\mathfrak{g} = \{X(\lambda) \in \mathfrak{gl}_{n|n}[\lambda] : \pi(X(\lambda)) = X(-\lambda)\}$$

acquires a natural Lie co-superalgebra structure. A quantization of the co-Poisson Hopf superalgebra $\mathrm{U}(\mathfrak{g})$ is constructed. An analogue of the Drinfeld duality theorem for the quantization of $\mathrm{U}(\mathfrak{sl}_n[\lambda])$ is also found.

1. Yangians. Let $\mathfrak{s}$ be a simple finite-dimensional Lie algebra over $\mathbf{C}$. Apart from $\mathrm{U}_q(\mathfrak{s})$, the quantized universal enveloping algebra of $\mathfrak{s}$, there is one more Hopf algebra to associate with $\mathfrak{s}$ in a natural way. This algebra, the *Yangian* $\mathrm{Y}(\mathfrak{s})$, plays a role of considerable importance in mathematical physics and representation theory [Ch,O1]. For the general description of $\mathrm{Y}(\mathfrak{s})$ in terms of generators and relations, see the original paper [D1]. Here I indicate only the classical counterpart of $\mathrm{Y}(\mathfrak{s})$, that is the polynomial current Lie algebra $\mathfrak{s}[\lambda]$ with a natural Lie bialgebra structure. To describe the latter, consider the following general situation.

Let $\mathfrak{a}$ be an arbitrary finite-dimensional Lie algebra and $K \in (\mathfrak{a}^{\otimes 2})^{\mathfrak{a}}$. Then the rational function

$$r(\lambda,\mu) = \frac{K}{\lambda-\mu} \tag{1.1}$$

obeys the classical Yang–Baxter equation

$$[{}^{12}r(\lambda,\mu), {}^{13}r(\lambda,\nu)] + [{}^{12}r(\lambda,\mu), {}^{23}r(\mu,\nu)] + [{}^{13}r(\lambda,\nu), {}^{23}r(\mu,\nu)] = 0; \tag{1.2}$$

here the standard tensor notation of [KS1] is used. If, additionally, $K \in \mathrm{S}^2\,\mathfrak{a}$ then $r(\lambda,\mu)$ is antisymmetric,

$$^{12}r(\lambda,\mu) + {}^{21}r(\mu,\lambda) = 0,$$

and the co-commutator $\varphi : \mathfrak{a}[\lambda] \to \mathfrak{a}[\lambda]^{\otimes 2} \cong \mathfrak{a}^{\otimes 2}[\lambda,\mu]$ can be defined by

$$\varphi(X(\lambda)) = [{}^1X(\lambda) + {}^2X(\mu), {}^{12}r(\lambda,\mu)]. \tag{1.3}$$

This definition of φ makes $\mathfrak{a}[\lambda]$ into a Lie bialgebra. In particular, if $\mathfrak{a}$ is simple and K is the Casimir element, one gets a natural Lie bialgebra structure on $\mathfrak{a}[\lambda]$.

The Yangian $\mathrm{Y}(\mathfrak{s})$ contains the universal enveloping algebra $\mathrm{U}(\mathfrak{s})$ as a subalgebra for any $\mathfrak{s}$. The case $\mathfrak{s} = \mathfrak{sl}_n$ is distinguished since only in this case a homomorphism $\mathrm{Y}(\mathfrak{s}) \to \mathrm{U}(\mathfrak{s})$ identical on $\mathrm{U}(\mathfrak{s})$ exists [D1]. In this section I will consider only this distinguished case.

To describe the Yangian $\mathrm{Y}(\mathfrak{sl}_n)$, consider the algebra $\mathrm{Y}(\mathfrak{gl}_n)$ with the generators $t_{ij}^{(k)}$ $(i,j = 1,\dots,n;\ k = 1,2,\dots)$ and the following relations. Introduce the formal power series

$$t_{ij}(u) = \delta_{ij} + t_{ij}^{(1)}u^{-1} + t_{ij}^{(2)}u^{-2} + \dots \tag{1.4}$$

in u^{-1} and form the matrix $T(u) = [t_{ij}(u)]_{i,j=1}^{n}$. Let $P \in \mathrm{End}((\mathbb{C}^n)^{\otimes 2})$ be the switch map; the $\mathrm{End}((\mathbb{C}^n)^{\otimes 2})$-valued function

$$R(u,v) = \mathrm{id} - \frac{P}{u-v}$$

obeys the quantum Yang–Baxter equation

$$^{12}R(u,v)\,^{13}R(u,w)\,^{23}R(v,w) = {}^{23}R(v,w)\,^{13}R(u,w)\,^{12}R(u,v). \tag{1.5}$$

Then

$$^{12}R(u,v)\,^{1}T(u)\,^{2}T(v) = {}^{2}T(v)\,^{1}T(u)\,^{12}R(u,v). \tag{1.6}$$

It is a direct consequence of (1.6) that the following formulae define comultiplication Δ, co-unit ε and antipode S on $\mathrm{Y}(\mathfrak{gl}_n)$:

$$\Delta(t_{ij}(u)) = \sum_k t_{ik}(u)\otimes t_{kj}(u), \quad \varepsilon(t_{ij}(u)) = \delta_{ij}, \quad S(t_{ij}(u)) = \tilde{t}_{ij}(u) \tag{1.7}$$

where $[\tilde{t}_{ij}(u)]_{i,j=1}^{n} = T^{-1}(u)$. The inclusion $\mathrm{U}(\mathfrak{gl}_n) \hookrightarrow \mathrm{Y}(\mathfrak{gl}_n)$ and the homomorphism $\mathrm{Y}(\mathfrak{gl}_n) \to \mathrm{U}(\mathfrak{gl}_n)$ identical on $\mathrm{U}(\mathfrak{gl}_n)$ can be defined, respectively, by

$$e_{ij} \mapsto t_{ij}^{(1)} \quad \text{and} \quad t_{ij}^{(k)} \mapsto 0,\ k \geq 2$$

where e_{ij} form the standard basis of $\mathfrak{gl}_n$.

By definition, the antipode S is an anti-automorphism of $\mathrm{Y}(\mathfrak{gl}_n)$, hence S^2 is an automorphism. Put

$$z(u) = \sum_i t_{ij}(u+n)\,\tilde{t}_{ji}(u) = 1 + z^{(1)}u^{-2} + z^{(2)}u^{-3} + \dots,$$

it does not depend on the index j.

PROPOSITION 1.1([D1,KS2]). *a)* $S^2(t_{ij}(u)) = z^{-1}(u)\,t_{ij}(u+n)$.

b) $\Delta(z(u)) = z(u) \otimes z(u)$, $S(z(u)) = z^{-1}(u)$.

c) The elements $z^{(1)}, z^{(2)}, \dots$ *generate the centre of* $\mathrm{Y}(\mathfrak{gl}_n)$.

The Hopf algebra $\mathrm{Y}(\mathfrak{sl}_n)$ is the quotient of $\mathrm{Y}(\mathfrak{gl}_n)$ by the relations

$$z^{(1)} = z^{(2)} = \cdots = 0.$$

Now I formulate a version of the Drinfeld duality theorem [D2], it provides a representation theory for the algebra $\mathrm{Y}(\mathfrak{gl}_n)$. The key role in [D2] is played by the *degenerate affine Hecke algebra* He_N. It is generated by the group algebra $\mathbb{C}[S_N]$ of the symmetric group and the pairwise commuting elements $x_1, \dots, x_N$ such that

$$\begin{aligned} &(a, a+1)\, x_b = x_b\, (a, a+1) \quad \text{if} \quad b \neq a, a+1; \\ &(a, a+1)\, x_a = x_{a+1}\, (a, a+1) - 1 \end{aligned} \tag{1.8}$$

where (a, b) is the transposition in S_N.

Consider the following general construction. Let G be a finite group and Xe be an assosiative algebra containing the group algebra $\mathbb{C}[G]$. Fix a representation σ of G in some vector space V and introduce the algebras

$$A = Xe \otimes \mathrm{End}(V), \qquad D = \bigoplus_{g \in G} \mathbb{C} \cdot g \otimes \sigma(g).$$

Denote by C the commutant of D in A and put $B = C \cdot D$. Then the left ideal L in B generated by the elements

$$1 \otimes \mathrm{id} - g \otimes \sigma(g), \quad g \in G$$

turns out to be a two-sided one; let $Y = B/L$ be the quotient algebra. For any Xe-module M, the action of B in the invariant space $(M \otimes V)^D$ is pulled through that of Y.

Let $G = S_N$ and σ be its action in $V = (\mathbb{C}^n)^{\otimes N}$ via permutations of the tensor factors: $\sigma((a, a+1)) = {}^{a,a+1}P$. Set $Xe = He_N$ and produce the algebra Y. Pass from the commuting generators x_a of He_N to the non-commuting ones,

$$u_a = x_a - \sum_{1 \le b < a} (a, b).$$

We get

$$(a, b) \cdot [u_a, u_b] = u_a - u_b,$$

but the relations (1.8) have been now purified:

$$s\, u_a\, s^{-1} = u_{s(a)}, \quad s \in S_N.$$

Evidently, one can define a homomorphism $He_N \to \mathbb{C}[S_N]$ trivial on S_N by

$$u_a \mapsto 0; \quad a = 1, \dots, N.$$

THEOREM 1.2. *The following map extends to a homomorphism* $\mathrm{Y}(\mathfrak{gl}_n) \to Y$:

$$ {}^{0}T(u) \mapsto 1 \otimes \mathrm{id} - \sum_{1 \leq a \leq N} \frac{1}{u - u_a} \otimes {}^{0a}P. \tag{1.9} $$

The proof proceeds by the direct substitution of the rhs of (1.9) into (1.6). This theorem clearly provides the functor

$$ \mathcal{D} : \{He\text{-modules}\} \to \{\mathrm{Y}(\mathfrak{gl}_n)\text{-modules}\} : M \mapsto (M \otimes (\mathbf{C}^n)^{\otimes N})^D. \tag{1.10} $$

Example. Let χ be a character of the commutative subalgebra $\mathbf{C}[x_1, \dots, x_N]$ of He_N. Consider the *principal series* He_N*-module*

$$ M_\chi = \mathrm{Ind}_{\mathbf{C}[x_1,\dots,x_N]}^{He_N} \chi. $$

Then the application of the functor $\mathcal{D}$ to M_χ is none other else than the *fusion process* of [KRS]:

$$ \mathcal{D}(M_\chi) = (\mathbf{C}^n)^{\otimes N}; \quad {}^{0}T(u) \mapsto {}^{0N}R(u, \chi(x_N)) \dots {}^{01}R(u, \chi(x_1)). $$

2. The queer Lie superalgebra. There are two analogues of the Lie algebra $\mathfrak{gl}_n$ in the theory of Lie superalgebras. The obvious counterpart is $\mathfrak{gl}_{n|m}$, and the generalization of the above constructions to that case is straightforward; cf. [N1]. The more interesting one is the *queer* Lie superalgebra $\mathfrak{q}_n$, the fixed point subalgebra in $\mathfrak{gl}_{n|n}$ with respect to the second-order automorphism

$$ \pi : e_{ij} \mapsto e_{i+n,j+n} $$

where e_{ij} are the standard generators of $\mathfrak{gl}_{n|n}$ and the indices i, j are considered mod $2n$.

The queerness of $\mathfrak{q}_n$ manifests itself in that all the invariants in $\mathrm{S}^2\, \mathfrak{q}_n$ are trivial:

$$ (\mathrm{S}^2\, \mathfrak{q}_n)^{\mathfrak{q}_n} = \mathbf{C} \cdot (e_{11} + \dots + e_{2n,2n})^{\otimes 2}. \tag{2.1} $$

Hence the co-commutator (2.1) vanishes and there is no natural Lie bi-superalgebra structure on $\mathfrak{q}_n[\lambda]$. But such a structure can be defined, in compensation, on the *twisted* current Lie superalgebra

$$ \mathfrak{g} = \{X(\lambda) \in \mathfrak{gl}_{n|n}[\lambda] : \pi(X(\lambda)) = X(-\lambda)\}. $$

The definition is based on the following general scheme (cf. [A,FR]).

Let $\mathfrak{a}, K$ be as above and ω be an automorphism of $\mathfrak{a}$ with finite order d. Let ζ be a primitive d-th root of unity. Define $r(\lambda, \mu)$ by means of (1.1) and put

$$ s(\lambda, \mu) = \sum_{c \in \mathbf{Z}/d\mathbf{Z}} (\mathrm{id} \otimes \omega^c)\, r(\lambda, \mu \cdot \zeta^c). \tag{2.2} $$

PROPOSITION 2.1. *Suppose that*

$$\omega^{\otimes 2}(K) = \zeta K, \tag{2.3}$$

then the function (2.2) *obeys the classical Yang–Baxter equation* (1.2). *If, in addition to* (2.3), $K \in \mathrm{S}^2\, \mathfrak{a}$ *then* (2.2) *is antisymmetric.*

Note that for an (even) simple Lie algebra $\mathfrak{a}$ we always have

$$\omega^{\otimes 2}(K) = K,$$

and in compliance with [BD1], our general construction gives nothing new in this case.

Now set $\mathfrak{a} = \mathfrak{gl}_{n|n}$, $\omega = \pi$ and

$$K = \sum_{i,j} e_{ij} \otimes e_{ji} \cdot (-1)^{p(j)}$$

where $p(i) = 0$ or 1 depending on whether $1 \le j \le n$ or $n+1 \le j \le 2n$. Then the condition (2.3) is satisfied and, moreover, $K \in \mathrm{S}^2\, \mathfrak{a}$. Therefore substituting $s(\lambda, \mu)$ for $r(\lambda, \mu)$ in (1.3) we get a Lie bi-superalgebra structure on $\mathfrak{g}$. The next section contains the description of certain Hopf superalgebra $\mathrm{Y}(\mathfrak{q}_n)$ which is a quantization of $\mathrm{U}(\mathfrak{g})$ in the sense of [D1].

3. Yangian of $\mathfrak{q}_n$. Consider the tautological $\mathfrak{gl}_{n|n}$-module $\mathbf{C}^{n|n}$. Let E_{ij} constitute the standard basis of the matrix superalgebra $\mathrm{End}(\mathbf{C}^{n|n})$. The action of the subalgebra $\mathfrak{q}_n \subset \mathfrak{gl}_{n|n}$ on $\mathbf{C}^{n|n}$ commutes with

$$I = \sum_j E_{j+n,j} \cdot (-1)^{p(j)}.$$

Let $P \in \mathrm{End}((\mathbf{C}^{n|n})^{\otimes 2})$ again be the switch map,

$$P = \sum_{i,j} E_{ij} \otimes E_{ji} \cdot (-1)^{p(j)}.$$

As a direct computation shows, the $\mathrm{End}((\mathbf{C}^{n|n})^{\otimes 2})$-valued rational function

$$R(u,v) = \mathrm{id} - \frac{P}{u-v} + \frac{P(I \otimes I)}{u+v}$$

obeys the quantum Yang–Baxter equation (1.5).

At this point, to describe the algebra $\mathrm{Y}(\mathfrak{q}_n)$ one can apply the standard technique of [FRT] which has been already used in Section 1. The algebra $\mathrm{Y}(\mathfrak{q}_n)$ is generated by the elements $t_{ij}^{(k)}$ $(i,j = 1,\dots,n;\ k = 1,2,\dots)$ subject to the following relations. Introduce the formal power series $t_{ij}(u)$ by means of (1.4) and form the matrix $T(u) = [t_{ij}(u)]_{i,j=1}^{2n}$. Then the relations imposed are (1.6) with the above $R(u,v)$, and

$$t_{ij}(-u) = t_{i+n,j+n}(u); \qquad i,j = 1,\dots,2n.$$

The relation (1.6) is applied here in its $\mathbf{Z}_2$-graded form [KS1]. So it can be instructive to rewrite (1.6) in a more detailed way:

$$
\begin{aligned}
&(u^2-v^2)[t_{ij}(u),t_{kl}(v)]\cdot(-1)^{(p(i)+p(j))\,p(k)} = \\
&(u+v)(t_{kj}(u)\,t_{il}(v)-t_{kj}(v)\,t_{il}(u))\cdot(-1)^{p(i)\,p(j)} - \\
&(u-v)(t_{k+n,j}(u)\,t_{i+n,l}(v)-t_{k,j+n}(v)\,t_{i,l+n}(u))\cdot(-1)^{(p(i)+1)\,(p(j)+1)}.
\end{aligned}
$$

The standard formulae (1.7) define a Hopf superalgebra structure on $\mathrm{Y}(\mathfrak{q}_n)$. Put

$$
z(u)=\sum_i t_{ij}(u)\,\tilde{t}_{ji}(u)\cdot(-1)^{p(i)+p(j)},
$$

it does not depend on the index j. Moreover $z(u)=z(-u)$, so

$$
z(u)=1+z^{(1)}u^{-2}+z^{(3)}u^{-4}+\dots
$$

for some $z^{(1)},z^{(3)},\dots\in\mathrm{Y}(\mathfrak{q}_n)$.

PROPOSITION 3.1([N2]). *a)* $S^2(t_{ij}(u))=z(u)^{-1}t_{ij}(u)$.
b) $\Delta(z(u))=z(u)\otimes z(u)$, $S(z(u))=z^{-1}(u)$.
c) The elements $z^{(1)},z^{(3)},\dots$ *generate the centre of* $\mathrm{Y}(\mathfrak{q}_n)$.

It follows from (2.1) that the Yangian of $\mathfrak{q}_n$ in the sense of [D1] does not exist. In the following two situations the Hopf superalgebra $\mathrm{Y}(\mathfrak{q}_n)$ plays the role of that non-existing Yangian, cf. Section 1.

Firstly, the inclusion $\mathrm{U}(\mathfrak{q}_n)\hookrightarrow\mathrm{Y}(\mathfrak{q}_n)$ and the homomorphism $\mathrm{Y}(\mathfrak{q}_n)\to\mathrm{U}(\mathfrak{q}_n)$ identical on $\mathrm{U}(\mathfrak{q}_n)$ can be defined respectively by

$$
e_{ij}+e_{i+n,j+n}\mapsto t_{ij}^{(1)}\cdot(-1)^{p(i)}\quad\text{and}\quad t_{ij}^{(k)}\mapsto 0,\ k\geq 2.
$$

Moreover, the images of $z^{(1)},z^{(3)},\dots$ under the latter homomorphism generate the centre of $\mathrm{U}(\mathfrak{q}_n)$ ([Se1]).

Secondly, there is an analogue of Theorem 1.2. Let Cl_N denote the *Clifford group* of range N, that is, the group generated by $c_1,\dots,c_N,\zeta$ subject to the relations

$$
\zeta^2=1;\quad c_a^2=\zeta;\quad c_a\,c_b=\zeta\,c_b\,c_a,\ a\neq b
$$

where ζ belongs to the centre. Consider the semi-direct product $S_N\ltimes Cl_N$:

$$
s\,c_a\,s^{-1}=c_{s(a)},\ s\,\zeta=\zeta\,s;\quad s\in S_N.
$$

Define the representation σ of this group in $(\mathbf{C}^{n|n})^{\otimes N}$ by

$$
\sigma(\zeta)=-\,\mathrm{id},\quad\sigma((a,a+1))={}^{a,a+1}P,\quad\sigma(c_a)={}^{a}I.
$$

PROPOSITION 3.2([Se2]). *The image of* $\mathrm{U}(\mathfrak{q}_n)$ *under the action on* $(\mathbb{C}^{n|n})^{\otimes N}$ *is equal to the commutant of* σ.

Introduce the *degenerate affine Sergeev algebra* Se_N, it is generated by the group algebra $\mathbb{C}[S_N \ltimes Cl_N]$ and the pairwise commuting elements $y_1, \dots, y_N$ such that

$$\begin{aligned} y_b\,(a,a+1) - (a,a+1)\,y_b &= 0 \quad \text{if } b \neq a, a+1; \\ y_{a+1}\,(a,a+1) - (a,a+1)\,y_a &= 1 - c_a\,c_{a+1}; \\ y_a\,c_b &= c_b\,y_a \quad \text{if } a \neq b; \\ y_a\,c_a &= -c_a\,y_a. \end{aligned}$$

Set $G = S_N \ltimes Cl_N$, $V = (\mathbb{C}^{n|n})^{\otimes N}$, $Xe = Se_N$ in the general construction of Section 1 and produce the algebra Y. As well as for He_N, pass from the commuting generators $y_1, \dots, y_N$ to the non-commuting ones,

$$v_a = y_a - \sum_{1 \le b < a} (1 - c_a c_b) \cdot (a, b).$$

Then we have

$$\begin{aligned} (a,b) \cdot [v_a, v_b] &= v_a - v_b + c_a\,c_b(v_a + v_b), \quad a \neq b; \\ s\,v_a\,s^{-1} &= v_{s(a)}, \quad s \in S_N; \\ v_a\,c_b &= c_b\,v_a \quad \text{if } a \neq b; \\ v_a\,c_a &= -c_a\,v_a. \end{aligned}$$

The homomorphism $Se_N \to \mathbb{C}[S_N \ltimes Cl_N]$ trivial on $S_N \ltimes Cl_N$ can be defined by

$$v_a \mapsto 0, \quad a = 1, \dots, N.$$

THEOREM 3.3. *The following map extends to a homomorphism* $\mathrm{Y}(\mathfrak{q}_n) \to Y$:

$${}^0T(u) \mapsto 1 \otimes \mathrm{id} - \sum_{1 \le a \le N} \frac{1}{u - v_a} \otimes {}^{0a}P + \sum_{1 \le a \le N} \frac{1}{u + v_a} \otimes {}^{0a}P\,{}^0I\,{}^aI.$$

Thus we get a functor parallel to (1.10):

$$\{Se_N\text{-modules}\} \to \{\mathrm{Y}(\mathfrak{q}_n)\text{-modules}\} : M \mapsto (M \otimes (\mathbb{C}^{n|n})^{\otimes N})^D. \tag{3.1}$$

The functor (3.1) has remarkable applications to such a classical subject as projective representations of the symmetric group S_N, see [N3] for details.

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